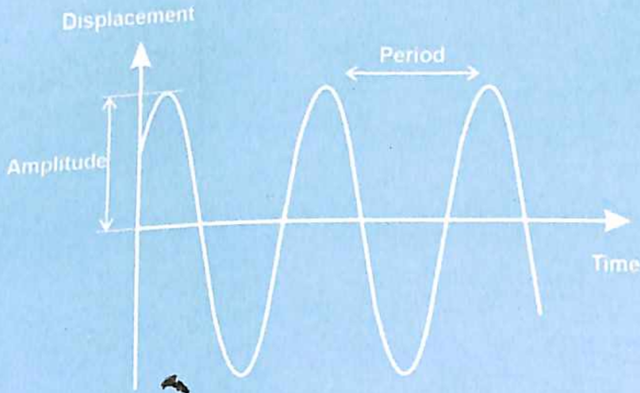
Waves and Oscillations



By
Dr. Prashant G. Gawali

According to revised CBCS syllabus of S. R. T. M. University, Nanded.

Waves and Oscillations

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Preface

The present book **Waves and Oscillations** is meant for B.Sc. Second Year degree course for all Indian Universities. The book has been written according to the latest syllabi prescribed by the Swami Ramanand Teerth Marathwada University, Nanded for B.Sc. Physics Second Yeae (CBCS) syllabus revised.

The book contains waves, free oscillations, damped and forced vibrations. Reverberation, Sabine's formula, production, detection and applications of ultrasonic's. This book is the pre-requisite for several advanced courses in Physics and is necessary for understanding the knowledge of elementary mathematics and calculus, wave theory etc. Specially the book will helps to the B.Sc. Second Year Physics students of university to understand concept in easy language. Every mathematical step is solved in the derivation.

I'm very grateful to my Physics teacher for free consultancy in preparing the manuscripts I'm also thankful to typist and publisher for presenting the book in present form.

Suggestions for further improvement of book are cordially invited.

*- **Prashnat Gawali***

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Differentiating equation (1) *w.r.t.* x

$$\frac{dy}{dx} = a \cdot \cos\frac{2\pi}{\lambda}(Vt - x) \cdot \frac{2\pi}{\lambda}$$

$$\frac{dy}{dx} = \frac{2\pi a}{\lambda} \cdot \cos\frac{2\pi}{\lambda}(Vt - x) \quad \ldots\ldots (3)$$

Put this value in equation (2)

$$\therefore\ \mathbf{U = V \cdot \frac{dy}{dx}} \quad \ldots\ldots (4)$$

Particle velocity at any instant = wave velocity × slope of displacement curve at that instant

1.2 Differential Equation of wave motion

The equation of displacement of a simple harmonic wave is given by

$$y = a \cdot \sin\frac{2\pi}{\lambda}(Vt - x) \quad \ldots\ldots (1)$$

Differentiating equation (1) *w.r.t.* t

$$\frac{dy}{dt} = \frac{2\pi a v}{\lambda} \cdot \cos\frac{2\pi}{\lambda}(Vt - x) \quad \ldots\ldots (2)$$

Again differentiating equation (2) *wr.t.* 't'

$$\frac{d^2y}{dt^2} = \frac{2\pi a}{\lambda} \times -\sin\frac{2\pi}{\lambda}(Vt - x) \times \frac{2\pi v}{\lambda}$$

$$\frac{d^2y}{dt^2} = -\frac{4\pi^2 v^2 a}{\lambda^2} \times \sin\frac{2\pi}{\lambda}(Vt - x) \quad \ldots\ldots (3)$$

Unit I

Waves

1.1 Wave Velocity and Particle Velocity

Particle velocity (U): The individual particles don't travel through medium simply oscillates about its mean position. The instantaneous velocity of particle is called particle velocity.

Wave velocity (V): The velocity with which the disturbance travelled through the medium is called wave velocity.

Relation between U & V:

The equation of displacement is given by,

$$y = a\sin\frac{2\pi}{\lambda}(Vt - x) \text{ ...... (1)}$$

where, a = amplitude, λ = wavelength, v = wave velocity, t = time

Differentiating equation (1) *w.r.t.* 't'

$$\frac{dy}{dt} = a\cdot\cos\frac{2\pi}{\lambda}(Vt - x)\cdot\frac{2\pi v}{\lambda}$$

$$U = \frac{dy}{dt} = \frac{2\pi a v}{\lambda}\cdot\cos\frac{2\pi v}{\lambda}(Vt - x) \text{...... (2)}$$

Differentiating equation (1) *w.r.t. x*

$$\frac{dy}{dx} = a\cdot\cos\frac{2\pi}{\lambda}(Vt-x)\cdot\frac{2\pi}{\lambda}$$

$$\frac{dy}{dx} = \frac{2\pi a}{\lambda}\cdot\cos\frac{2\pi}{\lambda}(Vt-x) \ldots\ldots (3)$$

Put this value in equation (2)

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$$\frac{dy}{dt} = \frac{2\pi av}{\lambda}\cdot\cos\frac{2\pi}{\lambda}(Vt-x) \ldots\ldots (2)$$

Again differentiating equation (2) *wr.t.* 't'

$$\frac{d^2y}{dt^2} = \frac{2\pi a}{\lambda}\times -\sin\frac{2\pi}{\lambda}(Vt-x)\times\frac{2\pi v}{\lambda}$$

$$\frac{d^2y}{dt^2} = -\frac{4\pi^2v^2a}{\lambda^2}\times\sin\frac{2\pi}{\lambda}(Vt-x) \ldots\ldots (3)$$

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Differentiating equation (1) *w.r.t.* 't'

$$\frac{dy}{dt} = a\cdot\cos\frac{2\pi}{\lambda}(Vt - x)\cdot\frac{2\pi v}{\lambda}$$

$$U = \frac{dy}{dt} = \frac{2\pi a v}{\lambda}\cdot\cos\frac{2\pi v}{\lambda}(Vt - x) \text{......} \; (2)$$

Where, a = amplitude

v = Velocity of wave

t = time

x = position of the wave

Differentiating above equation w.r.t. t

$$\frac{dy}{dt} = a\cdot\cos\frac{2\pi}{\lambda}(Vt - x)\cdot\frac{2\pi v}{\lambda}$$

$$U = \frac{2\pi a v}{\lambda}\cdot\cos\frac{2\pi}{\lambda}(Vt - x) \ldots\ldots (1)$$

U is the particle velocity.

The kinetic energy per unit volume of layer is given by,

KE/volume $= \frac{1}{2} \times$ mass $\times$ (particle velocity)^2

$= \frac{1}{2} \times m \times U^2 \ldots\ldots (2)$

But density = mass/ lin.volume

$\rho = m/\delta x$

$\therefore m = \rho\cdot \delta x \ldots\ldots (3)$

Putting value of m & U from equation (1) & (3), the equation (2) becomes,

$$KE = \frac{1}{2} \times \rho\cdot \delta x \times \left(\frac{2\pi a}{\lambda}\right)^2\cdot \cos^2\frac{2\pi}{\lambda}(Vt - x)$$

Total K.E. for whole length l is given by integrating above equation, Differentiating equation (1) *w.r.t. x*

$$\frac{dy}{dx} = a \cdot \cos\frac{2\pi}{\lambda}(Vt - x) \cdot \frac{2\pi}{\lambda}$$

$$\frac{dy}{dx} = \frac{2\pi a}{\lambda} \cdot \cos\frac{2\pi}{\lambda}(Vt - x) \;\ldots\ldots\; (4)$$

Again differentiating equation (4) *wr.t. x*

$$\frac{d^2y}{dx^2} = \frac{2\pi a}{\lambda} \times -\sin\frac{2\pi}{\lambda}(Vt - x) \times \frac{2\pi}{\lambda}$$

$$\frac{d^2y}{dx^2} = -\frac{4\pi^2 a}{\lambda^2} \times \sin\frac{2\pi}{\lambda}(Vt - x) \;\ldots\ldots\; (5)$$

Form equation (3) and (5)

$$\therefore \frac{d^2y}{dt^2} = V^2 \cdot \frac{d^2y}{dx^2} \;\ldots\ldots\; (6)$$

This is the equation of differential equation of wave motion.

1.3 Energy of Plane Progressive Wave

Progressive wave: The wave which continuously travels in a given direction.

The energy of wave is partly K.E. and partly P.E.

Mathematical treatment:

The equation of progressive wave is given by,

$Y = a \cdot \sin 2\pi / \lambda\, (Vt - x)$

$$\therefore \text{P.E.} = \tfrac{1}{2} \times \rho\, v^2 \times \left(\frac{dy}{dx}\right)^2 \times \delta x$$

But, $$\frac{dy}{dx} = -\frac{2\pi a}{\lambda} \cdot \cos \frac{2\pi}{\lambda}(Vt - x)$$

$$\therefore \text{P.E.} = \tfrac{1}{2} \times \rho\, v^2 \times \frac{4\pi^2 a^2}{\lambda^2} \cdot \cos^2 \frac{2\pi}{\lambda} \cdot (Vt - x) \times \delta x$$

Total P.E. is found by taking integration of above equation. After solving we get –

$$\textbf{P.E./volume} = \tfrac{1}{4} \times \rho \times \left(\frac{2\pi a v}{\lambda}\right)^2 \ldots\ldots (5)$$

The total energy/ volume is given by,

T.E. = K.E. + P.E.

Putting the equation (4) and (5)

$$\therefore \text{T.E.} = \tfrac{1}{4}\, \rho \left(\frac{2\pi a v}{\lambda}\right)^2 + \tfrac{1}{4}\, \rho \left(\frac{2\pi a v}{\lambda}\right)^2$$

$$\therefore \text{T.E.} = \tfrac{1}{2}\, \rho \left(\frac{2\pi a v}{\lambda}\right)^2$$

But, $v/\lambda = \eta$ = frequency

$$\therefore \text{T.E.} = \tfrac{1}{2}\, \rho\, 4\pi^2 a^2 \cdot x^2$$

$$\textbf{T.E.} = 2\pi^2 \rho\, a^2 x^2 \ldots\ldots (6)$$

Where, ρ = density of volume

a = amplitude

$$K.E. = \frac{1}{2} \times \rho \times \left(\frac{2\pi a v}{\lambda}\right)^2 \cdot \int_0^l \cos^2 \frac{2\pi}{\lambda}(Vt - x) \cdot \delta x$$

After solving we get,

$$K.E. = \frac{1}{2} \times \rho \times \left(\frac{2\pi a v}{\lambda}\right)^2 \times \frac{1}{2}$$

$$\textbf{K.E./volume} = \frac{1}{4} \times \rho \times \left(\frac{2\pi a v}{\lambda}\right)^2 \ldots\ldots (4)$$

Potential Energy:

The P.E. of layer per unit volume is given by

P.E. = work done × volume of layer (thickness)

$= \frac{1}{2} \times \text{stress} \times \text{strain} \times \delta x$

But, Elasticity $= \frac{stress}{strain}$

Stress = Elasticity × stain

$\text{Stress} = E \times \frac{dy}{dx}$

$\therefore P.E. = \frac{1}{2} \times E \times \frac{dy}{dx} \times \frac{dy}{dx} \times \delta x$

$= \frac{1}{2} E \left(\frac{dy}{dx}\right)^2 \times \delta x$

But, $v = \sqrt{\frac{E}{\rho}}$

$\therefore E = v^2 \rho$

$\therefore$ P.E. $= \frac{1}{2} \times \rho\, v^2 \times (\frac{dy}{dx})^2 \times \delta x$

But, $\frac{dy}{dx} = -\frac{2\pi a}{\lambda} \cdot \cos\frac{2\pi}{\lambda}(Vt - x)$

$\therefore$ P.E. $= \frac{1}{2} \times \rho\, v^2 \times \frac{4\pi^2 a^2}{\lambda^2} \cdot \cos^2\frac{2\pi}{\lambda} \cdot (Vt - x) \times \delta x$

Total P.E. is found by taking integration of above equation. After solving we get –

P.E./volume $= \frac{1}{4} \times \rho \times (\frac{2\pi a v}{\lambda})^2$ ...... (5)

The total energy/ volume is given by,

T.E. = K.E. + P.E.

Putting the equation (4) and (5)

$\therefore$ T.E. $= \frac{1}{4}\rho\,(\frac{2\pi a v}{\lambda})^2 + \frac{1}{4}\rho\,(\frac{2\pi a v}{\lambda})^2$

$\therefore$ T.E. $= \frac{1}{2}\rho\,(\frac{2\pi a v}{\lambda})^2$

But, $v/\lambda = \eta$ = frequency

$\therefore$ T.E. $= \frac{1}{2}\rho\, 4\pi^2 a^2 \cdot x^2$

T.E. $= 2\pi^2 \rho\, a^2 x^2$ ...... (6)

Where, ρ = density of volume

a = amplitude

$$K.E. = \frac{1}{2} \times \rho \times \left(\frac{2\pi a v}{\lambda}\right)^2 \cdot \int_0^l \cos^2 \frac{2\pi}{\lambda} (Vt - x) \cdot \delta x$$

After solving we get,

$$K.E. = \frac{1}{2} \times \rho \times \left(\frac{2\pi a v}{\lambda}\right)^2 \times \frac{1}{2}$$

$$\mathbf{K.E./volume} = \frac{1}{4} \times \rho \times \left(\frac{2\pi a v}{\lambda}\right)^2 \ldots\ldots (4)$$

Potential Energy:

The P.E. of layer per unit volume is given by

$$P.E. = \text{work done} \times \text{volume of layer (thickness)}$$

$$= \frac{1}{2} \times \text{stress} \times \text{strain} \times \delta x$$

But, Elasticity $= \frac{stress}{strain}$

$$\text{Stress} = \text{Elasticity} \times \text{stain}$$

$$\text{Stress} = E \times \frac{dy}{dx}$$

$$\therefore P.E. = \frac{1}{2} \times E \times \frac{dy}{dx} \times \frac{dy}{dx} \times \delta x$$

$$= \frac{1}{2} E \left(\frac{dy}{dx}\right)^2 \times \delta x$$

But, $v = \sqrt{\frac{E}{\rho}}$

$$\therefore E = v^2 \rho$$

Let us consider a small element PQ of displaced string having length δx is acted by two tensions T along the tangents PK and QR. Let Φ and $\Phi - \delta\Phi$ be the angles of the tangents to the curve at the ends of P and Q respectively. As every T (along PK & QR) have two components (i) Horizontal component (x-axis) (ii) Vertical component (y-axis). The Horizontal components cancelled with each other. The resultant component is the vertical components is given by,

$$\text{Force} = T \cdot d^2y/dx^2 \cdot \delta x \text{ .... (1)}$$

Where, $dy/dx = \text{Tan}\Phi = \text{slope}$

δx = length of small element string

According to Newton's 2nd law

Force = mass × acceleration

But mass per unit length

$$\therefore\ m = M/\delta x \ \therefore\ M = m \cdot \delta x$$

$$\text{And acceleration} = \frac{d^2y}{dt^2}$$

$$\therefore \text{force} = m \cdot \delta x \times \frac{d^2y}{dt^2} \text{ .... (2)}$$

From equation (1) and (2)

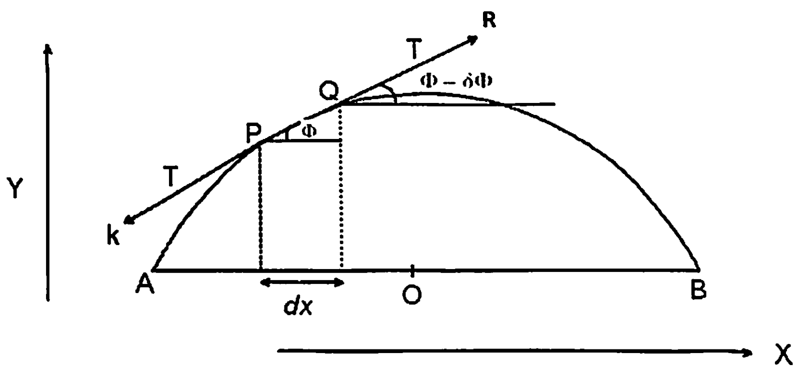
$$m \cdot \delta x \times \frac{d^2y}{dt^2} = T \times \frac{d^2y}{dx^2} \cdot \delta x$$

η = frequency of vibration

Equation (6) is the energy of progressive wave.

1.4 Equation of Motion of Vibrating String:

When any string stretched between two fixed points with a large tension. When plucked transverse and released then string vibrates to and fro along the plane perpendicular to the string. The transverse vibrations of a string have great importance to produce musical sounds in the instruments like guitar, violin and piano to produce music.



Consider the string AB is stretched between the points having tension T. The string is plucked at the centre O and released then it gets vibrations. It vibrates up and down, the string brings to its original position every time is due to the vertical component of the tension T right angles to AB. The string gets S.H.M.

Suppose the wave pulse travels along a stretched string from left to right.

Consider V → velocity of wave

PQ → is the small portion of string

δx → is the length of PQ

It forms a arc of circle having centre O, such that $\angle POQ = \theta$

R → Radius of the circle

M = mass of string

m = mass per unit length of string

$M = m \cdot \delta x$

T = tension in the string

The resultant force is given by,

$$F = T \cdot \theta \quad \text{.........} \quad (1)$$

This force F provides the centripetal force to the portion PQ and is given by,

$$\text{Centripetal force} = \frac{MV^2}{R}$$

$$F = \frac{MV^2}{R} \quad \text{.........} \quad (2)$$

$$\frac{d^2y}{dt^2} = \frac{T}{m} \times \frac{d^2y}{dx^2} \quad \ldots\ldots\ldots (3)$$

The differential equation of wave motion is,

$$\frac{d^2y}{dt^2} = V^2 \times \frac{d^2y}{dx^2} \quad \ldots\ldots\ldots (4)$$

Where, V = Velocity of wave travelling along a string.

Comparing equation (3) and (4)

$$V^2 = T/m$$

$$\mathbf{V} = \sqrt{\frac{T}{m}} \quad \ldots\ldots\ldots (5)$$

Where T = Tension in the string; m = $M/\delta x$ = mass per unit length of string.

Equation (5) is velocity of wave traveling along a vibrating string.

1.5 Velocity of Transverse Wave along A String

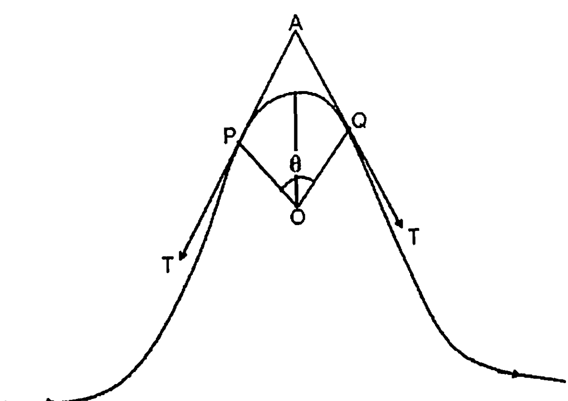


Fig.: Transverse wave in a stretched string

Formula: frequency = No. of cycles/second

Unit: It is measured in Hertz = Hz

Example: Any machine designed with the 50Hz or 60Hz frequency. It means the no. of cycles 50 or 60 completed in one second.

$$1KHz = 1000Hz = 10^3Hz = 1 \text{ Thousand}$$

$$1MHz = 1000000 = 10^6Hz = 1 \text{ Million}$$

$$1GHz = 1000000000 = 10^9Hz = 1 \text{ Billion}$$

The frequency at which a system tends to oscillate in the absence of any driving or damping force called natural frequency, it is called the normal mode.

Exp: According research, the natural frequency of a human body is about 7.5Hz

⇒ An object can have one or more natural frequencies. A system have one degree of freedom has 1 natural frequency system with 2 degree of freedom have 2 natural frequency.

Degree of freedom: The object which has free to vibrate or vary is called degree of freedom. The lowest frequency of vibration is called *fundamental frequency.*

$$V = n\lambda \qquad \therefore n = V/\lambda$$

From equation (1) and (2)

$$T\theta = \frac{MV^2}{R} \text{ .... } (3)$$

$$\text{But, } \theta = \frac{\text{arc length}}{\text{radius}} = \frac{\delta x}{R}$$

$$\text{And } M = m \cdot \delta x$$

Put these values in equation (3)

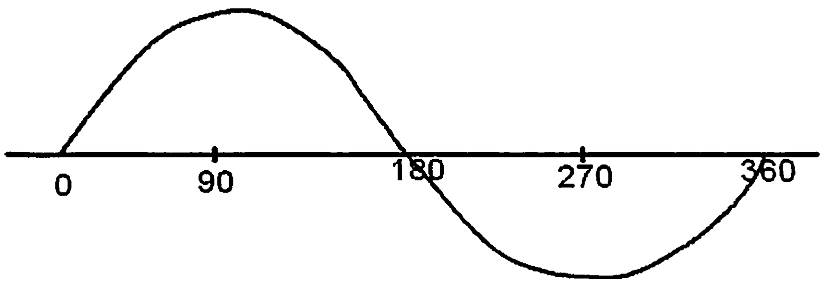
$$T \cdot \left(\frac{\delta x}{R}\right) = \frac{m \cdot \delta x \cdot V^2}{R}$$

$$T = m \cdot V^2$$

$$V^2 = \frac{T}{m}$$

$$V = \sqrt{\frac{T}{m}} \text{ .... } (4)$$

1.6 Frequency and Period of Vibration of String



Definition of Frequency: The number of cycles completed in one second is called the frequency of wave

1 cycle = one upper half part + lower half part

1.7 Exercises

Que. 1 Define wave and particle velocity.

Que. 2 Obtain relation between wave and particle velocity?

Que. 3 Obtain the differential equation of wave motion?

Que. 4 Give the mathematical treatment of total energy of plane progressive wave?

Que. 5 Derive the T.E. of plane progressive wave is $2\pi^2\rho a^2x^2$?

Que. 6 Obtain the equation of motion of velocity of vibrating string?

Que. 7 Derive the expression of velocity of transverse wave along a string?

Que. 8 Prove the $V = \sqrt{\frac{T}{m}}$ velocity of transeverse wave in string.

Que. 9 Explain the period and frequency of vibration in a string.

If $l \rightarrow$ length of string $\therefore \lambda = 2l$ $\therefore n = V/2l$

Put $V = \sqrt{\frac{T}{m}}$

$\therefore$ $\mathbf{n = 1/2l\sqrt{\frac{T}{m}}}$

Equation of fundamental frequency

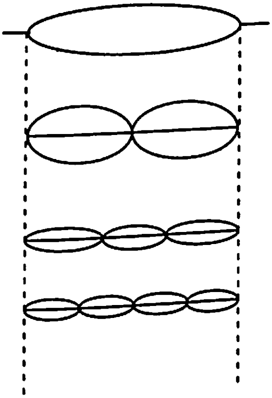
Period of Vibration

Definition: It is the time taken by an object to complete one cycle of vibration is called period or time period.

It is denoted by 'T'. "It is the reciprocal of frequency."

Formula: T = 1/F

Unit: Second



$$Y = a[2 \cdot \cos \frac{2\pi}{\lambda} \left(\frac{Vt - \cancel{x} + Vt + \cancel{x}}{2}\right) \cdot \sin \frac{2\pi}{\lambda} \left(\frac{\cancel{Vt} - x - \cancel{Vt} - x}{2}\right)]$$

$$Y = a[2 \cdot \cos \frac{2\pi Vt}{\lambda} \cdot \sin \frac{-2\pi}{\lambda}]$$

$$Y = -2a \sin \frac{2\pi x}{\lambda} \cdot \cos \frac{2\pi Vt}{\lambda} \text{ ...... (3)}$$

The resultant amplitude of particle is

$$A = 2a \sin \frac{2\pi}{\lambda} \text{ ...... (4)}$$

Taking derivative of equation (3) w.r.t. to 't'

$$\frac{dy}{dt} = -2a \sin \frac{2\pi x}{\lambda} \cdot (-\sin \frac{2\pi vt}{\lambda} \cdot \frac{2\pi v}{\lambda})$$

But, $\frac{dy}{dt} = V$ = Velocity of resultant stationary wave

$$\mathbf{V} = \frac{4\pi av}{\lambda} \cdot \sin \frac{2\pi x}{\lambda} \cdot \sin \frac{2\pi vt}{\lambda} \text{ ...... (5)}$$

Differentiating equation (3) w.r.t. to x

$$\frac{dy}{dx} = -2a \cos \frac{2\pi x}{\lambda} \cdot \frac{2\pi}{\lambda} \cdot \cos \frac{2\pi vt}{\lambda}$$

But $\frac{dy}{dx}$ = strain or compression

$$\therefore \textbf{Strain} = \frac{-4\pi a}{\lambda} \cdot \cos \frac{2\pi x}{\lambda} \cdot \cos \frac{2\pi vt}{\lambda} \text{ ...... (6)}$$

Changes with respect to position (x)

Unit II
STATIONARY WAVES

2.1 Closed End Organ Pipe or String Fixed at Other End:

The displacement of particle due to incident wave is given by,

$$Y_1 = a \cdot \sin \frac{2\pi}{\lambda}(Vt - x) \;\ldots\ldots\; (1)$$

Where, a = amplitude, λ = wavelength, v = velocity of wave in time 't'

The displacement of particle due to reflected wave is given by,

$$Y_2 = -\,a \cdot \sin \frac{2\pi}{\lambda}(Vt + x) \;\ldots\ldots\; (2)$$

The resultant displacement of particle

$$Y = Y_1 + Y_2$$

$$Y = [a \cdot \sin \frac{2\pi}{\lambda}(Vt - x) + (-\,a \cdot \sin \frac{2\pi}{\lambda}(Vt + x))]$$

$$Y = a[(\sin \underbrace{\frac{2\pi}{\lambda}(Vt - x)}_{A} - (\sin \underbrace{\frac{2\pi}{\lambda}(Vt + x)}_{B})]$$

But $\mathrm{Sin}A - \mathrm{Sin}B = 2\cos \frac{A+B}{2} \cdot \mathrm{Sin}\frac{A-B}{2}$

As strain is zero, these positions correspond to *antinodes*.

As $\sin \frac{2\pi}{\lambda} = \pm 1$

It means $\frac{2\pi x}{\lambda} = 0, \pi, 2\pi, \ldots\ldots$

$\therefore x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots\ldots$ etc.

Thus, the antinodes are equidistant and separated by $\frac{\lambda}{2}$

Changes with respect to time (*t*)

Case I: $\sin \frac{2\pi vt}{\lambda} = 0$ and $\cos \frac{2\pi vt}{\lambda} = \pm 1$

Put these values in equations (3), (4), (5), & (6) we get

Displacement $Y = \pm 2a \cdot \sin \frac{2\pi x}{\lambda}$

Amplitude $A = 2a \cdot \sin \frac{2\pi x}{\lambda}$ (*Not dependent on time*)

Velocity $V = 0$

Strain $\frac{dy}{dx} = \pm \frac{4\pi a}{\lambda} \cdot \cos \frac{2\pi x}{\lambda}$

Here, at these instants the displacement and strain are maximum but $V = 0$

As $\cos \frac{2\pi vt}{\lambda} = \pm 1$

$\frac{2\pi vt}{\lambda} = 0, \pi, 2\pi, \ldots\ldots$

Case I: $\sin\frac{2\pi x}{\lambda} = 0$ and $\cos\frac{2\pi}{\lambda} = \pm 1$

Put these values in equations (3), (4), (5), & (6) we get

Displacement $Y = 0$
Amplitude $A = 0$
Velocity $V = 0$
strain $\frac{dy}{dx} = \frac{-4\pi a}{\lambda} \cdot \cos\frac{2\pi vt}{\lambda}$
strain = maximum

So, these positions correspond to *nodes.*

As, $\sin\frac{2\pi x}{\lambda} = 0$

It means $\frac{2\pi x}{\lambda} = 0, \pi, 2\pi, \ldots\ldots$

$\therefore x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots\ldots$etc.

Thus, Nodes are equidistant and separated by $\frac{\lambda}{2}$

Case II: $\sin\frac{2\pi x}{\lambda} = \pm 1$ and $\cos\frac{2\pi x}{\lambda} = 0$

Put these values in equations (3), (4), (5), & (6) we get

Displacement $Y = \pm 2a \cdot \cos\frac{2\pi vt}{\lambda}$
Amplitude $A = \pm 2a$
Velocity $V = \pm\frac{4\pi a}{\lambda} \cdot \sin\frac{2\pi vt}{\lambda}$
Strain $\frac{dy}{dx} = 0$

The displacement of particle due to reflected wave is given by,

$$Y_2 = +\, a\cdot\sin\frac{2\pi}{\lambda}(Vt + x) \;\ldots\ldots\; (2)$$

The resultant displacement of particle

$$Y = Y1 + Y2$$

$$Y = [a\cdot\sin\frac{2\pi}{\lambda}(Vt - x)] + [a\cdot\sin\frac{2\pi}{\lambda}(Vt + x)]$$

$$Y = a[\underbrace{\sin\frac{2\pi}{\lambda}(Vt - x)}_{A} + \underbrace{\sin\frac{2\pi}{\lambda}(Vt + x)}_{B}]$$

But, $\mathrm{Sin}A + \mathrm{Sin}B = 2\sin\frac{A+B}{2}\cdot\cos\frac{A-B}{2}$

$$Y = a[2\cdot\sin\frac{2\pi}{\lambda}\left(\frac{Vt - \cancel{x} + Vt + \cancel{x}}{2}\right)\cdot\cos\frac{2\pi}{\lambda}\left(\frac{\cancel{Vt} - x - \cancel{Vt} - x}{2}\right)]$$

$$Y = a[2\cdot\sin\frac{2\pi Vt}{\lambda}\cdot\cos\frac{-2\pi x}{\lambda}]$$

$$Y = -2a\cos\frac{2\pi x}{\lambda}\cdot\sin\frac{2\pi Vt}{\lambda} \;\ldots\ldots\; (3)$$

The resultant amplitude of particle is

$$A = 2a\cos\frac{2\pi x}{\lambda} \;\ldots\ldots\; (4)$$

Taking derivative of equation (3) w.r.t. to 't'

$$\frac{dy}{dt} = -2a\cos\frac{2\pi x}{\lambda}\cdot(\cos\frac{2\pi vt}{\lambda}\cdot\frac{2\pi v}{\lambda})$$

$\therefore t = 0, \frac{T}{2}, T, \frac{3T}{2}, \ldots\ldots$etc.

At these instants, all particles are at *extreme position.*

Case II: $\sin\frac{2\pi vt}{\lambda} = \pm 1$ and $\cos\frac{2\pi vt}{\lambda} = 0$

Put these values in equations (3), (4), (5), & (6) we get

$$Y = 0,\ V = 0,\ \text{strain} = 0$$

As $\sin\frac{2\pi vt}{\lambda} = \pm 1$

$$\therefore \frac{2\pi vt}{\lambda} = 0, \pi, 2\pi, \ldots\ldots$$

We get $t = \frac{T}{4}, T, \frac{3T}{4}, \frac{5T}{4}\ldots\ldots$etc.

At these instants, all particles are at *mean position.*

It means in one period (T) particle passes two times through mean position.

2.2 Open End Pipe or String Free at the Other End

The displacement of particle due to incident wave is given by,

$$Y_1 = a \cdot \sin\frac{2\pi}{\lambda}(Vt - x) \ldots\ldots (1)$$

Where, a = amplitude, λ = wavelength, v = velocity of wave in time 't'

$\therefore x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots\ldots$etc.

Thus, Antinodes are equidistant and separated by $\frac{\lambda}{2}$

Case II: $\sin\frac{2\pi}{\lambda} = \pm 1$ and $\cos\frac{2\pi x}{\lambda} = 0$

Put these values in equations (3), (4), (5), & (6) we get

Displacement $Y = 0$
Amplitude $A = 0$
Velocity $V = 0$

$$\text{Strain } \frac{dy}{dx} = \pm \frac{4\pi a}{\lambda} \cdot \sin\frac{2\pi v}{\lambda}$$

These positions correspond to *nodes.*

As $\cos\frac{2\pi x}{\lambda} = 0$
It means $\frac{2\pi x}{\lambda} = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots\ldots$
$\therefore x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots\ldots$etc.

Thus, the nodes are equidistant and separated by $\frac{\lambda}{2}$

Changes with respect to time (*t*)

Case I: $\sin\frac{2\pi vt}{\lambda} = 0$ and $\cos\frac{2\pi vt}{\lambda} = 0$

Put these values in equations (3), (4), (5), & (6) we get
Displacement $Y = 0$

Amplitude $A = 2a \cdot \cos\frac{2\pi}{\lambda}$ (*Not dependent on time*)

But, $\frac{dy}{dt}$ = V = Velocity of resultant stationary wave

$$V = \frac{4\pi a v}{\lambda} \cdot \cos\frac{2\pi x}{\lambda} \cdot \cos\frac{2\pi v t}{\lambda} \;\ldots\ldots (5)$$

Differentiating equation (3) w.r.t. to x

$$\frac{dy}{dx} = -2a - \sin\frac{2\pi x}{\lambda} \cdot \frac{2\pi}{\lambda} \cdot \sin\frac{2\pi v t}{\lambda}$$

But, $\frac{dy}{dx}$ = strain or compression

$$\therefore \text{Strain} = \frac{4\pi a}{\lambda} \cdot \sin\frac{2\pi x}{\lambda} \cdot \sin\frac{2\pi v t}{\lambda} \;\ldots\ldots\ldots (6)$$

Changes with respect to position (x)

Case I: $\sin\frac{2\pi x}{\lambda} = 0$ and $\cos\frac{2\pi x}{\lambda} = \pm 1$

Put these values in equations (3), (4), (5), & (6) we get

Displacement $Y = \pm 2a \cdot \sin\frac{2\pi v t}{\lambda}$

Amplitude $A = \pm 2a$

Velocity $V = \pm\frac{4\pi a}{\lambda} \cdot \cos\frac{2\pi v t}{\lambda}$

Strain $\frac{dy}{dx} = 0$

As strain is zero, these positions correspond to *antinodes*.

As, $\sin\frac{2\pi x}{\lambda} = 0$

It means $\frac{2\pi x}{\lambda} = 0, \pi, 2\pi, \ldots\ldots$

2.3 Investigation of Pressure and Density Changes at Displacement Nodes & Antinodes:

Let us consider stationary wave is set up in the fluid, then excess pressure is given by,

$$P = -E\cdot\frac{dy}{dx} \quad \ldots\ldots\ldots\ldots\ldots (1)$$

Where, E = Volume Elasticity

$$E = v^2\rho \ldots\ldots\ldots\ldots\ldots (2)$$

V = velocity and ρ = density

$\frac{dy}{dx}$ = strain/compression

In case of open end pipe strain is given by,

$$\frac{dy}{dx} = \frac{4\pi a}{\lambda}\cdot\sin\frac{2\pi x}{\lambda}\cdot\sin\frac{2\pi vt}{\lambda} \ldots\ldots\ldots\ldots\ldots (3)$$

Putting equation (2) & (3) in equation (1) ....

$$P = v^2\rho\frac{4\pi}{\lambda}\cdot\sin\frac{2\pi x}{\lambda}\cdot\sin\frac{2\pi vt}{\lambda}$$

Put, $P_{max} = v^2\rho\frac{4\pi a}{\lambda} \ldots\ldots\ldots\ldots\ldots (4)$

$$\therefore P = P_{max}\cdot\sin\frac{2\pi x}{\lambda}\cdot\sin\frac{2\pi vt}{\lambda}$$

Put, $P_{max}\cdot\sin\frac{2\pi}{\lambda} = P_x \ldots\ldots\ldots\ldots\ldots (5)$

Velocity $V = \pm \frac{4\pi a}{\lambda} \cdot \cos\frac{2\pi x}{\lambda}$

Strain $\frac{dy}{dx} = 0$

Here, velocity is maximum.

As $\sin\frac{2\pi v}{\lambda} = 0$

$\frac{2\pi vt}{\lambda} = 0, \pi, 2\pi, \ldots\ldots$

$\therefore t = 0, \frac{T}{2}, T, \frac{3T}{2}, \ldots\ldots$etc.

It means in one period (T) particle passes two times through *mean position.*

Case II: $\sin\frac{2\pi vt}{\lambda} = \pm 1$ and $\cos\frac{2\pi vt}{\lambda} = 0$

Put these values in equations (3), (4), (5), & (6) we get

Displacement $Y = \pm 2a \cdot \cos\frac{2\pi x}{\lambda}$

Amplitude $A = 2a \cdot \cos\frac{2\pi x}{\lambda}$ *(independent on time)*

Velocity $V = 0$

Strain $\frac{dy}{dx} = \pm \frac{4\pi a}{\lambda} \cdot \sin\frac{2\pi x}{\lambda}$

As, $\sin\frac{2\pi vt}{\lambda} = \pm 1$

$\therefore \frac{2\pi vt}{\lambda} = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots\ldots$

$\therefore t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4} \ldots\ldots$etc.

At these instants, all particles are at *extreme position.*

Conclusion:

1. At nodes, there is maximum variation of pressure at position or displacement nodes.
2. There is minimum (zero) variation of pressure at position or displacement antinodes.
   Hence,
   Position or displacement node = pressure antinodes.
   Position or displacement antinodes = pressure nodes.

2.4 Distribution of Energy in a Stationary Wave

1. The distribution of energy in a stationary wave is not the same as in progressive wave. The total energy in a stationary wave is double that in a progressive wave.
2. As in stationary wave, one is incident and another is reflected wave in opposite direction. These two waves are equal but opposite, hence resultant transfer of energy is zero.
3. In progressive wave, at any moment half energy is kinetic and other half is potential.

In stationary wave, when particle moving through their equilibrium positions. All energy is kinetic everywhere and most of energy is near the antinodes and particle is moving fastest. But when particle is at extreme, K.E. is zero and all energy is potential everywhere. In the intermediate stages it is partly in one form and partly in other form.

Equation (5) is Amplitude of pressure

$$\therefore P = P_x \cdot \sin\frac{2\pi vt}{\lambda} \text{ .... (6)}$$

***Case I*: At Antinode**: Strain is zero

$$\frac{dy}{dx} = 0, \sin\frac{2\pi}{\lambda} = 0$$

Equation (5) and (6) becomes

$$P_x = 0 \quad \therefore P = 0$$

"Hence, at antinodal points the pressure and density are zero; it means no changes of pressure and density, remains normal."

***CaseII*: At Nodes:** Strain is maximum

$$\text{i.e. } \frac{dy}{dx} = \text{maximum; and } \therefore \sin\frac{2\pi x}{\lambda} = \pm 1$$

Equation (5) and (6) becomes

$$P_x = P_{max} \text{ and } P = \pm P_{max} \cdot \sin\frac{2\pi vt}{\lambda}$$

Here, change in pressure is maximum.

***Case III*: At any point**

The equation (i) is given by,

$$P = P_{max} \cdot \sin\frac{2\pi x}{\lambda} \cdot \sin\frac{2\pi vt}{\lambda} \text{ .... (7)}$$

Putting value of equations (1) and (2)….

$$k/Area = ½ ρ \int_0^l v^2 dx$$

$$k/Area = ½ ρ \int_0^l \left(\frac{4\pi av}{\lambda}\right)^2 \cdot \cos^2\frac{2\pi x}{\lambda} \cdot \cos^2\frac{2\pi vt}{\lambda} \cdot dx$$

$$= ½ ρ \cdot 16 \left(\frac{\pi av}{\lambda}\right)^2 \cdot \cos^2\frac{2\pi vt}{\lambda} \int_0^l \cos^2\frac{2\pi x}{\lambda} \cdot dx$$

$$= ρ \times 4 \times 2 \left(\frac{\pi av}{\lambda}\right)^2 \cdot \cos^2\frac{2\pi vt}{\lambda} \int_0^l \cos^2\frac{2\pi}{\lambda} \cdot dx$$

$$= ρ \left(\frac{2\pi av}{\lambda}\right)^2 \cdot \cos^2\frac{2\pi vt}{\lambda} \int_0^l 2\cos^2\frac{2\pi x}{\lambda} \cdot dx$$

But $2\cos^2\theta = 1 + \cos2\theta$

$$= ρ \left(\frac{2\pi av}{\lambda}\right)^2 \cdot \cos^2\frac{2\pi v}{\lambda} \int_0^l 1 + \cos\frac{4\pi x}{\lambda} \cdot dx$$

As, $[\therefore \int_0^l (1 + \cos\frac{4\pi x}{\lambda} \cdot dx = l]$

$$k/Area = ρ \left(\frac{2\pi av}{\lambda}\right)^2 \cdot \cos^2\frac{2\pi v}{\lambda} \cdot l$$

$$\therefore \frac{k}{Area \cdot l} = ρ \left(\frac{2\pi av}{\lambda}\right)^2 \cdot \cos^2\frac{2\pi vt}{\lambda}$$

$$\frac{K.E.}{volume} = ρ \left(\frac{2\pi av}{\lambda}\right)^2 \cdot \cos^2\frac{2\pi vt}{\lambda} \text{ .... (3)}$$

K.E./volume = maximum if $\cos^2\frac{2\pi vt}{\lambda} = \pm1$ = T.E.

$$\frac{T.E.}{volume} = ρ \left(\frac{2\pi av}{\lambda}\right)^2 \text{............. (4)}$$

Analytical Treatment:

In case of open end pipe the resultant displacement is,

$$Y = 2a \cdot \cos\frac{2\pi x}{\lambda} \cdot \sin\frac{2\pi v}{\lambda}$$

Differentiating above equation w.r.t. time 't'

$$\frac{dy}{dx} = 2a \cdot \cos\frac{2\pi x}{\lambda} \cdot \cos\frac{2\pi vt}{\lambda} \cdot \frac{2\pi v}{\lambda}$$

$$V = \frac{4\pi a}{\lambda} \cdot \cos\frac{2\pi x}{\lambda} \cdot \cos\frac{2\pi vt}{\lambda} \text{ .... (1)}$$

Let, $\rho \rightarrow$ density of medium

$dx \rightarrow$ thickness of layer

$$\text{density} = \frac{\text{mass}}{\text{volume}} = \frac{\text{mass}}{\text{thickness}}$$

$$\rho = \frac{\text{mass}}{dx}$$

$$\therefore \text{mass} = \rho \cdot dx \text{ .... (2)}$$

K.E. per unit area of layer is given by,

$$\frac{dk}{Area} = \frac{1}{2} \times \text{mass} \times (\text{velocity})^2$$

K.E. for whole wave of length '*l*' is given by taking integration...

$$\int dk = {}_0\int^l \frac{1}{2} \times \text{mass} \times (v)2$$

Compare equation of K.W. (3) and equation P.E. (7) and equation T.E. (4)

Case I:

Put, $\cos^2 \frac{2\pi v t}{\lambda} = \pm 1$ in equation (3)

Then K.E. = T.E.

Case II:

Put, $\sin^2 \frac{2\pi v t}{\lambda} = \pm 1$ or $\cos^2 \frac{2\pi}{\lambda} = 0$

$\therefore$ P.E. = T.E.

i.e. when K.E. = 0, P.E. = 0

when P.E. = 0, K.E.= 0

2.5 Energy is not transferred in stationary waves

The work done of energy transferred per unit area in a small time 'dt' is given by –

Energy transferred

= change in pressure × change in velocity × time

$= P \times U \times dt$ .... (1)

But, $P = P_x \sin \frac{2\pi v t}{\lambda}$ .... (2)

$U = U_x \cos \frac{2\pi v t}{\lambda}$ .... (3)

Putting these values in equation (1)

This is the average total energy per unit volume of stationary wave.

$$\therefore \frac{T.E.}{volume} = \rho \left(\frac{2\pi av}{\lambda}\right)^2 \ldots\ldots\ldots\ldots (5)$$

But T.E. per unit volume of progressive wave is given by,

$$\frac{T.E.}{volume} = \frac{1}{2} \rho \left(\frac{2\pi av}{\lambda}\right)^2 \ldots\ldots\ldots\ldots (6)$$

Compare equation (5) and (6),

T.E. of stationary wave is double the T.E. of progressive wave hence the proof.

But,

P.E. = T.E. – K.E.

P.E. = equation (5) – equation (3)

$$P.E. = \rho \left(\frac{2\pi av}{\lambda}\right)^2 - \rho \left(\frac{2\pi av}{\lambda}\right)^2 \cdot \cos^2 \frac{2\pi vt}{\lambda}$$

$$P.E. = \rho \left(\frac{2\pi av}{\lambda}\right)^2 \left[1 - \cos^2 \frac{2\pi vt}{\lambda}\right]$$

But, $1 - \cos^2\theta = \sin^2\theta$

$$P.E. = \rho \left(\frac{2\pi av}{\lambda}\right)^2 \cdot \sin^2 \frac{2\pi vt}{\lambda}$$

$$\therefore P.E. = \rho \left(\frac{2\pi av}{\lambda}\right)^2 \cdot \sin^2 \frac{2\pi vt}{\lambda} \ldots\ldots\ldots\ldots (7)$$

2.6 Exercises

Que. 1 Give analytical treatment of stationary wave for closed end organ pipe?

Que. 2 Obtain expressions for displacement, velocity and strain for stationary waves pipe closed at one end and hence what are changes with respect to position and time?

Que. 3 Give analytical treatment for stationary wave for open end pipe?

Que. 4 Obtain expressions for stationary wave for open end pipe?

Que. 5 Prove that antinodes of displacement are the nodes of pressure and vice versa?

Que. 6 Investigate pressure and density changes at displacement nodes and antinodes?

Que. 7 Obtain expression for K.E. and P.E. per unit volume for stationary wave?

Que. 8 Prove that average total energy of a stationary wave at any instant is double that of progressive wave.

Que. 9 Prove mathematically energy is not transferred in a stationary wave?

$$\text{Energy Transferred} = P_x \sin\frac{2\pi vt}{\lambda} \cdot U_x \cos\frac{2\pi v}{\lambda} \cdot dt$$

The total energy transferred for the periodic time 'T' is given by integrating above equation

$$\text{Energy Transferred} = \int_0^T P_x \sin\frac{2\pi vt}{\lambda} \cdot U_x \cdot \cos\frac{2\pi vt}{\lambda} \cdot dt$$

Multiplying both sides by $\frac{1}{T}$

$$\frac{\text{Energy transferred}}{T} = \frac{1}{T}\int_0^T P_x \sin\frac{2\pi vt}{\lambda} \cdot U_x \cdot \cos\frac{2\pi vt}{\lambda} \cdot dt$$

Multiplying and dividing by 2 in R.H.S. of equation

$$\text{Rate of energy} = \frac{P_x U_x}{2T}\int_0^T 2\sin\frac{2\pi vt}{\lambda} \cdot \cos\frac{2\pi vt}{\lambda} \cdot dt$$

But $2\sin\theta \cdot \cos\theta = \sin 2\theta$

$$\therefore \text{Rate of energy} = \frac{P_x U_x}{2T}\int_0^T \sin\frac{4\pi vt}{\lambda} \cdot dt$$

As, $\int_0^T \sin\frac{4\pi vt}{\lambda} \cdot dt = 0$

$\therefore$ Rate of energy = 0

Hence, the no energy is transferred, hence the proof.

general, practically there is always some resistance to be overcome which is called *damping*.

3.2 Undamped Vibrations:

In case of simple harmonic motion the kinetic energy of displacement 'y' is given by,

$$K.E. = \frac{1}{2} \times \text{mass} \times (\text{velocity})^2$$

$$K.E. = \frac{1}{2} \times m \times \left(\frac{dy}{dt}\right)^2$$

At the same instant the particle has potential energy –

$$P.E. = \frac{1}{2} Ky^2$$

Where, K – restoring force per unit displacement

Then total energy = K.E. + P.E.

$$= \frac{1}{2} m \left(\frac{dy}{dt}\right)^2 + \frac{1}{2} Ky^2 \;\ldots\ldots\ldots\ldots (1)$$

For undamped vibration T.E. = constant

Differentiating equation (1) w.r.t. to time

$$\frac{1}{2} \times m \times 2 \frac{d^2y}{dt^2} + \frac{1}{2} \times K \times 2 \frac{dy}{dt} = 0$$

Dividing by m,

$$\frac{d^2y}{dt^2} + \left(\frac{K}{m}\right) y = 0 \;\ldots\ldots\ldots\ldots (2)$$

Unit III
FREE AND FORCED VIBRATIONS

3.1 Free Vibrations:

Free vibrations of the body is kind of vibration when a force is exerted on the body only at once and the body starts to vibrate at its natural frequency.

Example: (1) When a bob of pendulum (in vacuum) is displaced from its *mean position*, it gets simple harmonic motion. Such vibrations are called *free vibrations*.

(2) Vibrations of tuning fork.

Free vibrations depend upon:

1. Dimensions of body
2. Elastic constant of material of which body is made.
3. As pendulum gets vibration, the amplitude period are depends upon the length of pendulum and gravity.

When body vibrating freely has no resistance offered to its motion, its *amplitude* remains constant. In such cases loss of energy is zero vibrations are also called *undamped free vibration* or *natural vibrations*. And the frequency of vibration is called *natural frequency*. But in

- When the vibration the energy is lost or dissipated in the form of heat within system or in surrounding medium.
- This dissipative force due to friction is proportional to the velocity of particle at that instant.

The differential equation for free undamped vibration is given by –

$$m \times \frac{d^2y}{dt^2} + Ky = 0 \quad \ldots\ldots\ldots\ldots (1)$$

Let $\mu \cdot \frac{dy}{dt}$ is the dissipative force due to friction then the differential equation for damped vibration is the addition of dissipative force in undamped.

$$\therefore m \times \frac{d^2y}{dt^2} + Ky + \mu \cdot \frac{dy}{dt} = 0 \quad \ldots\ldots\ldots\ldots (2)$$

$$\frac{d^2y}{dt^2} + \left[\frac{\mu}{m}\right] \cdot \frac{dy}{dt} + \left[\frac{K}{m}\right] y = 0 \quad \ldots\ldots\ldots\ldots (3)$$

This is the differential equation. The general differential equation is –

$$\frac{d^2y}{dt^2} + 2b \cdot \frac{dy}{dt} + K^2y = 0 \quad \ldots\ldots\ldots\ldots (4)$$

Comparing equation (3) and (4)

$$2b = \frac{\mu}{m} ; b = \frac{\mu}{2m} \text{ and } K^2 = \frac{K}{m}$$

The solution of this equation (3) is,

This is the differential equation of undamped free vibration. But we know,

$$\frac{d^2y}{dt^2} + \omega^2 y = 0 \text{ .... } (3)$$

Comparing equation (2) and (3)

$$\omega^2 = \frac{K}{m}$$

The solution of equation (3) is given,

$$y = a\sin(\omega t - \alpha)$$

$$y = a\sin\left[\sqrt{\frac{K}{m}} \cdot t - \alpha\right]$$

The frequency of oscillation is given by,

$$f = \frac{\omega}{2\pi}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}} \text{ .... } (4)$$

3.3 Damped Vibrations:

In actual practice, when the pendulum vibrates in air medium, there are frictional forces and energy is lost in each vibration. So the amplitude of swing is decreases and time increases. Finally the amplitude becomes zero or die out. Such vibrations are called *free damped vibrations*.

by pressing key. Then discharged through L and R when key is released.

Let Q = charge on condenser

$I = \text{current} = \frac{dQ}{dt}$

$\frac{dI}{dt} = \frac{d^2Q}{dt^2}$ = Rate of fall of current.

In this case force equation is replaced by voltage equation.

$$\frac{Q}{C} + RI + L\frac{dI}{dt} = 0$$

Put, $\frac{dI}{dt} = \frac{d^2Q}{dt^2}$, $I = \frac{dQ}{dt}$ in above equation.

$$L\cdot\frac{d^2Q}{dt^2} + R\cdot\frac{dQ}{dt} + \frac{Q}{C} = 0$$

$$\frac{d^2Q}{dt^2} + \frac{R}{L}\cdot\frac{dQ}{dt} + \frac{Q}{LC} = 0 \text{ .... (1)}$$

Put $\frac{R}{L} = 2b$ and $\frac{Q}{LC} = k^2$

$$\frac{d^2Q}{dt^2} + 2b\frac{dQ}{dt} + k^2Q = 0$$

The general solution of this equation is –

$$Q = A\cdot e^{(-b+\sqrt{b^2-K^2})t} + B\cdot e^{(-b-\sqrt{a^2-K^2})t} \text{ .... (2)}$$

When $t = 0$ and $Q = Q_0$

$$y = a \cdot e^{-bt} \cdot \sin(\omega t - \alpha) \;\ldots\ldots\ldots\ldots\; (5)$$

The general solution of equation (3) is given by,

$$y = A \cdot e^{(-b + \sqrt{b^2 + K^2})t} + B \cdot e^{(-b - \sqrt{a^2 - K^2})t}$$

$$\text{And } \omega = \sqrt{k^2 - b^2}$$

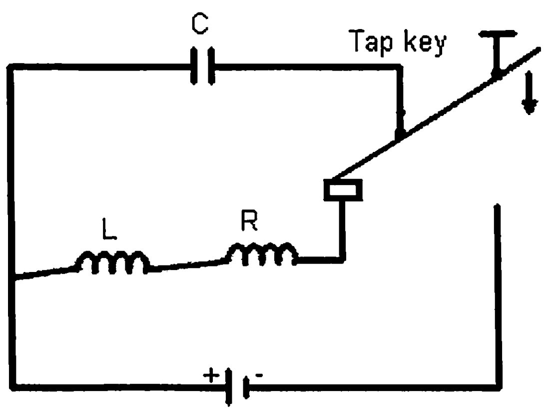
The frequency is given by,

$$f = \frac{\omega}{2\pi}$$

$$f = \frac{1}{2\pi} \cdot \sqrt{k^2 - b^2}$$

Where, $K^2 = \frac{K}{m}$ and $b = \frac{\mu}{2m}$

3.4 Damped SHM in an Electrical Circuit:



Let us consider the L (inductor), C (capacitor), R (resistor) circuit. The condenser C is charged due to battery

$$2A = Q_0 \left[1 + \frac{b}{\sqrt{b^2 - K^2}}\right]$$

$$A = \frac{Q_0}{2} \left[1 + \frac{b}{\sqrt{b^2 - K^2}}\right] \quad \ldots\ldots\ldots\ldots (5)$$

Subtracting equation (3) – (4)

$$A + B - A + B = Q_0 - \frac{bQ_0}{\sqrt{b^2 - K^2}}$$

$$2B = Q_0 \left[1 - \frac{b}{\sqrt{b^2 - K^2}}\right]$$

$$B = \frac{Q_0}{2} \left[1 - \frac{b}{\sqrt{b^2 - K^2}}\right] \quad \ldots\ldots\ldots\ldots (6)$$

Putting these values of 'A' and 'B' in equation (2)

$$Q = \frac{Q_0}{2} \left[1 + \frac{b}{\sqrt{b^2 - K^2}}\right] \cdot e^{(-b + \sqrt{b^2 - K^2})t} +$$

$$\frac{Q_0}{2} \left[1 - \frac{b}{\sqrt{b^2 - K^2}}\right] \cdot e^{(-b - \sqrt{b^2 - K^2})t}$$

But, $b = \frac{R}{2L}$ and $k = \sqrt{\frac{1}{LC}}$

Putting values of 'b' and 'k' in above equation,

$$Q = \frac{Q_0}{2} \left[1 + \frac{\frac{R}{2L}}{\sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}}\right] \cdot e^{(\frac{-R}{2L} + \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}) \cdot t} +$$

$\therefore A + B = Q_0$ .... (3)

Differentiating equation (2)

$$\frac{dQ}{dt} = A\cdot(-b + \sqrt{b^2 - K^2}) \cdot e^{(-b+\sqrt{b^2-K^2})t} +$$

$$B\cdot(-b - \sqrt{a^2 - K^2})t \cdot e^{(-b-\sqrt{a^2-K^2})t}$$

$$\text{At } t = 0, \frac{dQ}{dt} = 0$$

$$\therefore A\cdot(-b + \sqrt{b^2 - K^2}) + B\cdot(-b - \sqrt{a^2 - K^2})t = 0$$

$$A \times -b + A\cdot \sqrt{b^2 - K^2} + B \times -b \times -B \cdot \sqrt{b^2 - K^2} = 0$$

$$-bA - bB + A\sqrt{b^2 - K^2} - B\sqrt{b^2 - K^2} = 0$$

$$-b(A + B) + \sqrt{b^2 - K^2}\,(A - B) = 0$$

But, from equation $A + B = Q_0$

$$\therefore -bQ_0 + \sqrt{b^2 - K^2}\,(A - B) = 0$$

$$\sqrt{b^2 - K^2}\,(A - B) = bQ_0$$

$$\therefore (A - B) = \frac{bQ_0}{\sqrt{b^2 - K^2}} \text{ .... (4)}$$

Adding equation (3) + (4)

$$A + B + A - B = Q_0 + \frac{bQ_0}{\sqrt{b^2 - K^2}}$$

Then the discharge is *non oscillatory* and is said to be dead beat shown as curve 'b' in figure.

***Case III*:** When $\frac{R^2}{4L^2} < \frac{1}{LC}$

The square roots are imaginary.

Then the discharge is *oscillatory* shown as curve 'c' in figure. It is simple harmonic type. In this case, the natural frequency is –

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \quad \ldots\ldots\ldots\ldots (9)$$

When R = 0,

$$f = \frac{1}{2\pi\sqrt{LC}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

3.5 Forced Vibrations:

If some external periodic force is constantly applied on the body, it continues to oscillate under the influence of external forces. Such vibrations of the body are called *forced vibrations*.

Initially, the amplitude of swing increases, then decreases with time, becomes minimum and again increases. This will be repeated if the external force is constantly applied on the system. Finally the body will vibrate with the same frequency as that of applied force.

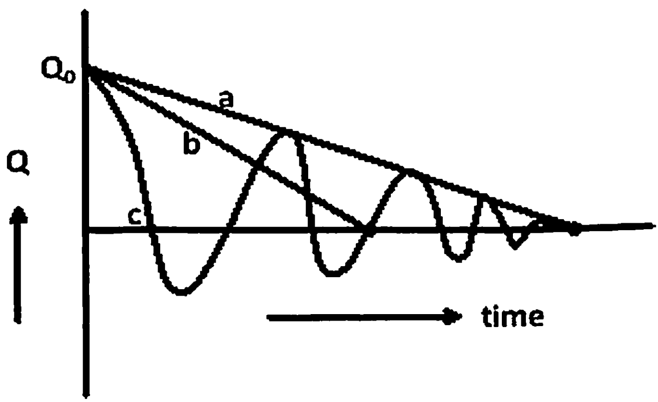
$$\left[1 - \frac{\frac{R}{2L}}{\sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}}\right] \cdot e^{-(\frac{-R}{2L} - \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}})t} \quad \ldots (7)$$

Special Cases:

Case I**:** When $\frac{R^2}{4L^2} = \frac{1}{LC}$

The equation (7) becomes

$$Q = Q_0 \cdot e^{\frac{-Rt}{2L}}$$



Then the discharge is *aperiodic* and critically damped. The exponential curve is shown as 'a' in the figure.

Case II**:** When $\frac{R^2}{4L^2} > \frac{1}{LC}$

The square roots are real.

$$-mp^2a\cdot\cos\alpha + ka\cdot\cos\alpha + \mu ap\cdot\sin\alpha - F = 0 \quad \ldots\ldots\ldots (6)$$

When, cospt = 1; sinpt = 0 equation (5) becomes –

$$-mp^2a\cdot\sin\alpha + ka\cdot\sin\alpha + \mu ap\cdot\cos\alpha = 0$$

$$\therefore mp^2a\cdot\sin\alpha - ka\cdot\sin\alpha - \mu ap\cdot\cos\alpha = 0 \quad \ldots\ldots\ldots (7)$$

Dividing (7) by cosα

$$mp^2a\cdot\tan\alpha - ka\cdot\tan\alpha - \mu ap\cdot\frac{1}{\tan\alpha} = 0$$

$$\tan\alpha\left[mp^2a - ka - \mu ap\cdot\frac{1}{\tan^2\alpha}\right] = 0$$

After simplifying,

$$\operatorname{Tan}\alpha = \frac{\mu p}{k-mp^2}$$

Put, $\mu p = A$ and $k - mp^2 = B$

$$\therefore \tan\alpha = \frac{A}{B} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

From equation (8)

$$\operatorname{Sin}\alpha = \frac{A}{\sqrt{A^2+B^2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

$$\operatorname{Cos}\alpha = \frac{B}{\sqrt{A^2+B^2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

Dividing equation (6) by cosα –

$$mp^2a + ka + \mu ap\cdot\tan\alpha - \frac{F}{\cos\alpha} = 0$$

The amplitude of vibration of a body is equal to the difference between the natural frequency and frequency of applied force. If difference is small, then amplitude will be large.

For Forced vibration the equation is,

$$m\frac{d^2y}{dt^2} + ky + \mu\frac{dy}{dt} = F\cdot \sin\cdot pt \;\ldots\ldots\ldots\ldots\ldots\ldots\; (1)$$

Where, p is the angular frequency of applied force. But,

$$y = a\sin(pt - \alpha) \;\ldots\ldots\ldots\ldots\ldots\ldots\; (2)$$

$$\frac{dy}{dt} = ap\cdot \cos(pt - \alpha) \;\ldots\ldots\ldots\ldots\ldots\ldots\; (3)$$

$$\frac{d^2y}{dt^2} = -ap^2\sin(pt - \alpha)$$

$$\frac{d^2y}{dt^2} = -p^2y \;\ldots\ldots\ldots\ldots\ldots\ldots\; (4)$$

Putting these values in equation (1)

$$-mp^2a\cdot\sin(pt - \alpha) + ka\cdot\sin(pt - \alpha) + \mu\cdot ap\cdot\cos(pt - \alpha) = F\cdot\sin\cdot pt$$

$$\Rightarrow -mp^2a\cdot[\sin pt\cdot\cos\alpha - \cos pt\cdot\sin\alpha]$$

$$+ ka\cdot[\sin pt\cdot\cos\alpha - \cos pt\cdot\sin\alpha]$$

$$+ \mu ap[\cos pt\cdot\cos\alpha + \sin pt\cdot\sin\alpha] - F\cdot\sin\cdot pt = 0 \;\ldots\ldots\; (5)$$

When, $\sin pt = 1 \therefore \cos pt = 0$

Thus,

General solution = solution of free vibration + solution of forced vibration

$$\therefore y = a \cdot e^{-bt} \cdot \sin(\omega t - \alpha) + \frac{F}{\sqrt{\mu^2 p^2 + (k - mp)^2}} \cdot \sin(pt - \alpha)$$

.... (14)

Here, $b = \mu/2m$

3.6 Resonance and Sharpness of Resonance:

When the frequency of periodic applied force (force vibration) is equal to the natural frequency of the system, then system vibrates with maximum amplitude the phenomenon is called as *Resonance*.

Types:

1. Mechanical resonance
2. Acoustic resonance
3. Electromagnetic resonance
4. Nuclear Magnetic Resonance (NMR)
5. Electron Spin Resonance (ESR)
6. Electric Resonance

Derivation of Response (R)

(i) Equation of amplitude at Resonance:

The amplitude of forced vibration is given by,

$$a\cdot[k - mp^2 + \mu p\cdot \tan\alpha] = \frac{F}{\cos\alpha}$$

but $k - mp^2 = B$ and $\mu p = A$;
Put values of $\tan\alpha$ and $\cos\alpha$ –

$$a\cdot[B + A \times \frac{A}{B}] = \frac{F\sqrt{A^2+B^2}}{B}$$

$$a\cdot[B + \frac{A^2}{B}] = \frac{F\sqrt{A^2+B^2}}{B}$$

$$a\cdot[\frac{A^2+B^2}{\cancel{B}}] = \frac{F\sqrt{A^2+B^2}}{\cancel{B}}$$

$$a = \frac{F\cancel{\sqrt{A^2+B^2}}}{\cancel{\sqrt{A^2+B^2}}\cdot\sqrt{A^2+B^2}}$$

$$\therefore \quad a = \frac{F}{\sqrt{A^2+B^2}}$$

Putting values of A and B,

$$a = \frac{F}{\sqrt{\mu^2p^2+(k-mp)^2}} \text{ .... } (11)$$

Put this value of a in the equation (2)

$$\therefore y = \frac{F}{\sqrt{\mu^2p^2+(k-mp)^2}}\cdot \sin(pt-\alpha) \text{ .... } (12)$$

This is for forced vibration, we know for free vibration,

$$y = a\cdot e^{-bt}\cdot \sin(\omega t-\alpha) \text{ .... } (13)$$

$$y = \frac{F}{\sqrt{\mu^2 p^2 + (k - mp^2)^2}} \cdot \sin(pt - \alpha) \quad \ldots\ldots\ldots\ldots\ldots (1)$$

differentiating above equation (1) *w.r.t.* time

$$\frac{dy}{dt} = \frac{FP}{\sqrt{\mu^2 p^2 + (k - mp^2)^2}} \cdot \cos(pt - \alpha) \quad \ldots\ldots\ldots\ldots (2)$$

For $\left(\frac{dy}{dt}\right)_{max}$, $\cos(pt - \alpha) = max. = 1$

$$\therefore \left(\frac{dy}{dt}\right)_{max} = \frac{FP}{\sqrt{\mu^2 p^2 + (k - mp^2)^2}}$$

This is velocity maximum,

$$V_{max} = \frac{FP}{\sqrt{\mu^2 p^2 + (k - mp^2)^2}} \quad \ldots\ldots\ldots\ldots (3)$$

$$K.E. = ½ \times m \times V^2_{max}$$

$$K.E. = \frac{\frac{1}{2} m F^2 P^2}{\mu^2 p^2 + (k - mp^2)^2} \quad \ldots\ldots\ldots\ldots (4)$$

The mean square driving force per unit mass is –

$$= \frac{\frac{(\text{Initial force}) + (\text{Final Force})^2}{2}}{m}$$

$$= \frac{\frac{0 + F^2}{2}}{m}$$

$$= \frac{F^2}{2m} \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Dividing equation (4) by equation (5)

$$\frac{(K.E.)_{max}}{\frac{F}{2m}} = \frac{\frac{1}{2} m F^2 P^2}{\mu^2 p^2 + (k - mp^2)^2}$$

$$\text{amplitude} = +\frac{F}{\sqrt{\mu^2 p^2 + (k - mp)^2}} \quad \text{.................} \quad (1)$$

where, μ = Frictional force

p = angular frequency of forced vibration

(Amplitude) maximum then denominator term is minimum. So $k - mp^2 = 0$

$$\therefore k = mp^2$$

$$\therefore k = \sqrt{\frac{k}{m}}$$

$$(\text{Amplitude})_{max} = \sqrt{\frac{F}{\mu p}}$$

Putting value of p $\quad \therefore (\text{Amp.})_{max} = \frac{F}{\mu\sqrt{\frac{k}{m}}}$

$$\therefore (\text{Amp.})_{max} = \frac{F}{\mu} \cdot \sqrt{\frac{m}{k}} \quad \text{.................} \quad (2)$$

This state of vibration of a system is called as *Resonance*.

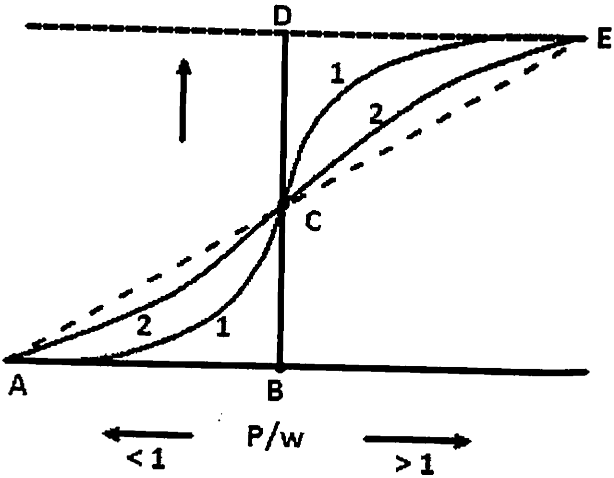
$$\therefore (\text{Amp.})_{\text{at Resonance}} = \frac{F}{\mu} \cdot \sqrt{\frac{m}{k}}$$

(ii) Equation of Response

The displacement for forced vibration is,

2. When $\mu = 0$, then R is infinite and sharpness of resonance is maximum.
3. As μ frictional force increases, then sharpness of resonance or response is decreases.
4. For curve A, μ is large. For curve C, μ is small.

3.7 Phase of Resonance:



The equation of phase of forced vibrations with reference to the driving force is given by –

$$\tan\alpha = \frac{\mu p}{k - mp^2}$$

Dividing numerator & denometer by 'm'

$$\tan\alpha = \frac{\frac{\mu p}{m}}{\frac{k}{m} - mp^2}$$

t is called as response R

$$\therefore R = \frac{P^2}{\frac{\mu^2 p^2}{m^2} + \left[\frac{k}{m} - P^2\right]^2} \text{ .... (6)}$$

R will be maximum, denominator is minimum i.e. $\frac{k}{m} = P^2$

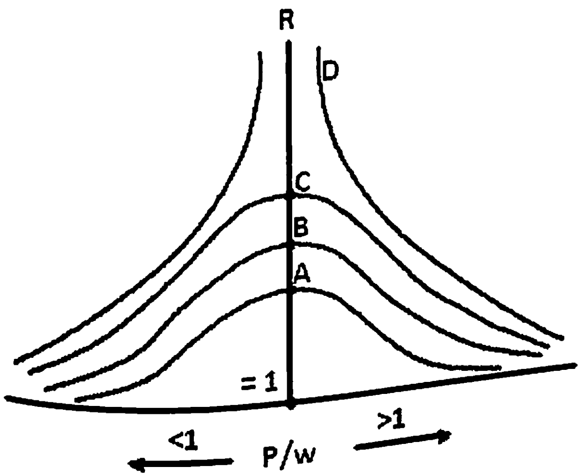
$$\therefore (R)max = \frac{P^2}{\frac{\mu^2 p^2}{m^2}}$$

$$\therefore (R)max = \left(\frac{m}{\mu}\right)^2 \text{ .... (7)}$$

As m is constant,

$\therefore$ Response R α $1/\mu$

"Response is inversely proportional to the frictional force (μ)".



In fig., p/ω is along x-axis and response R is along y-axis

1. When $p/\omega = 1$, then response is maximum and when $p/\omega < 1$ or > 1 then response decreases.

(Washington USA 1940 bridge collapse due to 42miles/hwind)

2. A child applies the external periodic force to set the swing to and fro. The amplitude of swing gradually increases. When the natural frequency of swing is equal to forced frequency by child, then resonance occurs and swing oscillates with a large amplitude.
3. In case sonometer, the paper rider on string is thrown off, when the forced frequency of tunning fork is equal to natural frequency of sonometer wire due to resonance occurs and wire vibrates with maximum amplitude.

3.9 Exercises

Que. 1 What are free vibrations, factors on which it depends?

Que. 2 Obtain equation of frequency for undamped vibration?

Que. 3 Prove $\omega = \sqrt{\frac{K}{m}}$ in case of undamped vibration?

Que. 4 Obtain differential equation of undamped vibration?

Que. 5 Derive the differential equation of damped oscillatory motion and give its general solution.

Que. 6 Deduce the equation of charge (Q) for damped SHM in an electrical circuit?

Que. 7 What are the conditions of discharge as aperiodic, non oscillatory and oscillatory in a damped SHM?

Que. 8 What is mean by forced vibrations?

At resonance $P^2 = k/m$

$$\therefore \tan\alpha = \infty$$

$$\therefore \alpha = \pi/2$$

It means,

$p/w = 1, \alpha = \pi/2$

$p/w > 1, \alpha > \pi/2$

$p/w < 1, \alpha < \pi/2$

From the graph,

1. For values of $\mu = 0$, the curve is ABCDE
2. For large values of μ, the curve is along 2
3. For small values of μ, the curve is along 1

3.8 Examples of Forced and Resonant Vibrations:

1. When soldier's are marching or stepping over a suspension bridge, then periodic stepping vibrations are forced to the hanging bridge, then the bridge swings with higher amplitude. When the natural frequency of bridge and forced frequency of marching is equal then the resonance occurs and bridge vibrates or swings with highest amplitude and it is dangerous to collapse the bridge. So the soldier troops have ordered to stop marching when crossing over the hanging bridge.

Unit IV
ACOUSTICS AND ULTRASONIC

Introduction:

Acoustics is the branch of physics that deals with the process of generation, reception and propagation of sound.

It is study of mechanical waves in gases, liquids and solids including topics such as vibration, sound ultrasound and infrasound.

Acoustics looks first at the pressure levels and frequencies in the sound wave and how the wave interacts with the environment. This interaction can be described as either diffraction, interference or a reflection or a mix of three.

The entire spectrum cab be divided into three sections: Audio, Ultrasonic and Infrasonic. The audio range falls between 20Hz to 20,000Hz, detected by human ear. Ultrasonic range is higher than 20,000Hz (shorter wavelength). It has better resolution in imaging technologies used as ultra sonography. The lower frequency than 20Hz are infrasonic.

Field of Acoustics:

1. Designing of acoustical instrument

Que. 9 Obtain the general solution for free and forced vibrations?

Que. 10 What is mean by Resonance and state types of Resonance?

Que. 11 Obtain equation of amplitude of resonance?

Que. 12 Explain Graphically the response is inversely proportional to frictional force.

Que. 13 Give the examples of forced and resonant vibrations?

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2. "The reverberation time of a room is defined as the time takes for sound to decay by 60dB." e.g. if the sound in a room took 10 seconds to decay from 100dB to 40dB, the reverberation time is 10 sec. T_{60}."

The reverberation time is important in defining how a room will respond to acoustic sound. In a good auditorium it is necessary to keep the reverberation time negligibly small.

For clear audibility of speech, it needs –

(1) Each separate note has intensity in every part of room.

(2) Each note dies rapidly before next note is heard by listener.

4.3 Derivation of Reverberation Time (Sabine's Formula)

Let,

- s_1, s_2, s_3 ... are the absorbing surfaces in a room
- a_1, a_2, a_3 ... are the absorption coefficients at reach reflection
- $S = s_1 + s_2 + s_3$ ... is the total area of surfaces

$\bar{a}$ is the average value of absorption coefficient is given by,

$$\bar{a} = \frac{a_1 s_1 + a_2 s_2 + a_3 s_3 + \ldots\ldots}{s_1 + s_2 + s_3 + \ldots\ldots}$$

2. Electro-acoustics ⇒ methods of sound production and recording e.g. microphones, loudspeakers, amplifiers
3. Architectural acoustics: Design and construction of building, operas, music hall, recording rooms in radio and television broadcasting stations.
4. Musical acoustics: Design of musical instruments.

4.1 Reverberation:

"The persistence of sound after its source has stopped, caused by multiple reflections of the sound within a closed space."

The persistence of sound in a big hall due to repeated reflections from walls, ceiling, floor of the hall is called *reverberation.*

In big hall excessive reverberation is highly undesirable. If reverberation is too long, then sound becomes blurred, distorted and confusing due to overlapping of different sounds.

4.2 Reverberation time:

1. "The time gap between the initial direct note (sound) and the reflected sound (note) up to the minimum audibility level is called *reverberation time.*"

   It depends upon (a) size of a room (b) Nature of material of wall and ceiling (c) Area of reflecting surface.

$$\therefore 10^{-6} = (1 - \bar{a})^{\frac{svt}{4v}}$$

Taking log of both sides

$$\log_e 10^{-6} = \log_e (1 - \bar{a})^{\frac{svt}{4v}}$$

$$\log_e 10^{-6} = \frac{svt}{4v} \cdot \log_e (1 - \bar{a})$$

$$\frac{svt}{4v} = \frac{\log_e 10^6}{\log_e (1-\bar{a})}$$

$$T = \frac{4v}{sv} \cdot \frac{\log_e 10^6}{\log_e (1-\bar{a})}$$

$$T = \frac{4 \times 2.303 \times \log 10^6 \times V}{v\, s \log_e (1-\bar{a})}$$

Put v = 1120 ft/sec.

$$T = \frac{2.303 \times 4 \times -6 \times V}{1120 \times s \log_e (1-\bar{a})}$$

$$T = \frac{0.05 \times V}{-s \log_e (1-\bar{a})} \text{ .... (3)}$$

This is the Eyring's formula.

As $\bar{a}$ is usually less than 1. Take $\log_e (1 - \bar{a})$ and expand series expansion

$$\therefore \log_e (1 - \bar{a}) = [-\bar{a}, -\bar{a}^2/2, -\bar{a}^3/2, - \ldots\ldots]$$

$$\bar{a} = \frac{\Sigma a_1 s_1}{S}$$

$$\therefore \Sigma a_1 s_1 = \bar{a}s \;\ldots\ldots\ldots\ldots\; (1)$$

Let, I_0 = Initial intensity of sound

I_t = Intensity of sound after time 't'

V = volume of room

v = velocity of sound

$\bar{a}$ = at single absorption the fraction absorbed

4V/S = distance between two successive reflections given by Jager –

∴ Velocity = dist./time

$$v = \frac{\frac{4V}{S}}{t} \qquad \therefore v = \frac{4V}{St}$$

∴ In time 't' the average number of reflection is $\frac{svt}{4v}$

The intensity of sound after time 't' is given by –

$$I_t = I_0 \left(1 - \bar{a}\right)^{\frac{svt}{4v}} \;\ldots\ldots\ldots\ldots\; (2)$$

But by definition of reverberation period

If t = period of reverberation = T

$$\frac{I_t}{I_0} = 10^{-6}$$

Put value of I_t

$$a = \frac{\text{energy of sound absorbed by a surface}}{\text{total sound energy incident on the surface}}$$

Sabine's Formula: $T = \frac{kV}{A}$

T = Reverberation time

K = Proportionality constant = 0.165

V = Volume of room

A = Effective absorpting surface area of walls and materials inside the room

T = (1) Directly proportional to V

(2) Inversely proportional to A

Professor W. C. Sabine (1868-1919) of Harvard University studied the whole subject of architectural acoustics scientifically. It was essential to choose a standard of absorption in terms of which the absorption by all substances can be measured. Sabine choose 1sq.ft. of an open window as a standard unit of absorption, since all sound waves falling on it pass through and can be said to be completely absorbed. This unit has been named as Sabin.

4.5 Acoustic Measurements:

Measurements of the values that describe sounds and noises in terms of their intensities. The principal values measured in acoustics are sound pressure, sound intensity etc. Human ear responds to sound wave pressure. For sound

As $\bar{a} \ll 1$ so the terms $\bar{a}^2/2$, $\bar{a}^3/2$ higher terms can be neglected.

$\therefore \log_e (1 - \bar{a}) = - \bar{a}$ ...put in equation (3)

$$\therefore T = \frac{0.05 \times V}{- s \times - \bar{a}}$$

$$\therefore T = \frac{0.05\ V}{s\bar{a}} \text{ ..... (In foot) ....(4)}$$

The expression,

$$T = \frac{k V}{A}$$

Here in fps system k = 0.05

If v volume of room is in m^3 and S Area of surface is in m^2 *then equation (4) becomes –*

$$T = \frac{0.165\ V}{s\bar{a}} \text{ ..... (In meter) ....(5)}$$

Here constant k = 0.165

$$\mathbf{T = \frac{0.165\ V}{\Sigma a_1 S_1}}$$

4.4 Absorption Coefficient

Absorption co-efficient (a) of any material defined as "the ratio of energy of sound absorbed by the surface to the total sound energy incident on the surface."

(1) Acoustic Intensity Level:

$$I.L. = 10 \log_{10} I/I_0 \text{ db}$$

But, $I_0 = 10^{-12}$ watt/m^2

$$I.L. = 10 \log_{10} I/10^{-12} \text{ db}$$

$$I.L. = 10 \log (I \times 10^{12}) \text{ db}$$

$$I.L. = 10 [\log I + \log 10^{12}] \text{ db}$$

$$I.L. = 10 \log I + 10 [12 \log 10] \text{ db}$$

$$I.L. = [10 \log I + 120] \text{ db}$$

(2) Acoustic Pressure Level:

$$P.L. = 10 \log_{10} [P/P_0]^2 \text{ db}$$

But, $P_0 = 2 \times 10^{-5}$ N/m^2

$$P.L. = 10 \log [P/2\times 10^{-5}]^2 \text{ db}$$

$$P.L. = 10 \log [P \times 0.5 \times 10^5]^2 \text{ db}$$

$$P.L. = 2 \times 10 [P \times 0.5 \times 10^5] \text{ db}$$

$$P.L. = [20 \log P + 20 \log 5 \times 10^4] \text{ db}$$

$$P.L. = [20 \log P + 20 \times 4.699] \text{ db}$$

$$P.L. = [20 \log P + 94] \text{ db}$$

4.6 Conditions for Good Acoustical Design of Auditorium:

(1) *Control of Reverberation:*

- A few open windows, the entire sound energy out serve as perfect absorbers.

pressure measurements a standard microphone is used in air or a hydrophone in water, ther received signal or pressures converted into electrical signal (voltages). The sound level meter is used for measuring various noises.

Intensity of sound is often measured as the ratio with the standard intensity I_0. But generally the logarithmic scale is used for measuring acoustic intensity, acoustic power and acoustic pressure. It is observed acoustic intensity corresponding to an energy 10^{-12} watt/m^2 is *audible* and it represents zero level of intensity or *reference intensity* I_0.

$$\text{Intensity level (I.L.)} = \log_{10} I/I_0$$

If sound intensity is 10 times than I_0 then –

$$\text{I.L.} = I/I_0 = 10 = 1 \text{ bel}$$

$$\text{I.L.} = I/I_0 = 10^2 = 2 \text{ bel}$$

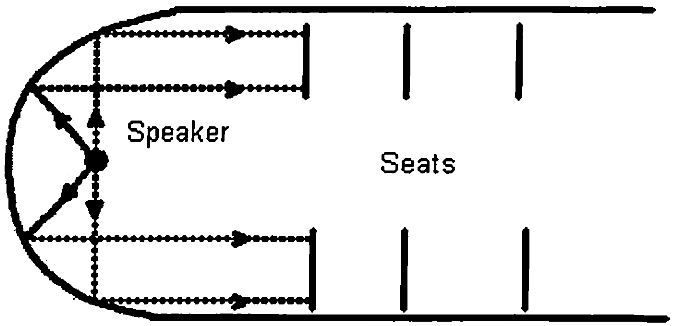
$$\text{I.L.} = I/I_0 = 10^3 = 3 \text{ bel}$$

The unit of intensity level is known as *bel*, named in honiur of Alexender Graham Bell.

In practice smller unit which is 1/10 bel us used and is known as *decibel* (dB).

$$\text{I.L.} = 10 \log_{10} I/I_0 \text{ db}$$

$$\therefore 1 \text{ bel} = 10\text{db}$$



The speaker is at the focus of the parabolic reflecting surface. The seats are gradually elevated in order to heard direct sound from source.

(5) Floor plan should have with diverging side walls so that sound will reach to those seats where the sound level is not adequate.

(6) *Echelon effect*:

If regular successive echo's of sound reaching the observer, an effect is called *echelon effect*. To avoid it the stair cases are covered with carpet for reflection of sound.

(7) *Noise*:

It is due to sound received from outside, sound of fans, air circulation in an auditorium.

⇒ Increase double triple windows and a.c. pipes should be covered with corks.

- Walls covered with absorbent materials by hanging heavy curtains, pictures and maps.
- Porous bodies by providing small capillaries absorbs sound energy incident on them.
- By having food audience. Each person is equivalent to about $0.5m^2$ area of an open window.

(2) *No cylindrical or spherical surfaces of walls and ceiling*:

There should no curved walls or corners bounded by two walls, it gives rise to undesirable focusing, avoid the focusing and concentration of sound at one place.

(3) *Concave surfaces and balconies*:

The sound in auditorium must be distributed over the whole area. The domes, curved arches should be avoided for echo's, focusing effect and non distribution of sound.

(4) *Seats*: For good distribution of sound, the seas are upholstered so that absorption is approximately the same with or without the audience.

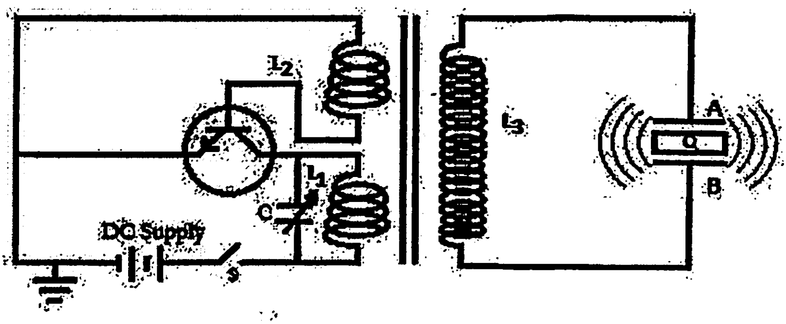
4.8 Piezoelectric Oscillator

Piezo electric effect:

When a mechanical compression or 5tension or pressure is applied to some crystals like quartz, a potential difference is developed across the crystal. This phenomenon is known as *piezo-electric effect.*

Also, if the potential difference is applied across the crystal, a mechanical pressure or compression or tension is developed this is called *inverse piezo electric effect.*

e.g. slices of crystal of quartz, tourmaline and Rochellel salts.



***Principle*:** This based on the inverse piezo electric effect. When a quartz crystal is subjected to an alternating potential difference, crystal is set into electric oscillations. If frequency of vibration of crystal is equivalent to natural

4.7 Ultrasonic's

- The audible frequency is 20Hz to 20,000Hz. It is audible range of humans.
- "Vibrations of frequencies greater than the upper limit of the audible range for humans i.e. greater than about 20kHz. The term sonic is applied as *Ultrasonic's*.
- The wavelength of ultrasonic waves are small e.g. at room temp. f = 20,000Hz and v = 350 m/s/
  $\therefore v = n\lambda \Rightarrow \lambda = v/n = 350/20000 = 0.0175m$
- The frequencies lower than 20Hz are called *infrasonic*.
- The ultrasonic waves cannot travel through vacuum.
- They undergoes reflection, refraction and absorption.
- They produces the intense heat when passed through objects.
- Ultrasonic waves are sound waves i.e. longitudinal waves that produce alternate compressions and rarefactions.
- Animals like dolphins and bats rely on ultrasound wave to live. Bats makes the sound have ultrasonic, these sound gets transmitted and bat receives echo, time interval between transmitted and echo received, as well as angle they find the distance and target.

P = 1, 2, 3 ......etc. i.e. 1st overtone, 2nd overtone etc. respectively;

$$f = \frac{1}{2\pi\sqrt{L_1 C_1}}$$

***Advantages*:**

- Ultrasonic frequency as high as 500 MHz can be generated.
- It is more efficient than the magnetostriction oscillator.
- The output power is not affected by temperature humidity.
- It gives stable and constant frequency.

***Disadvantages*:**

- The cost of quartz crystal is very high.
- Cutting and shaping of crystal is quite complex.

4.9 Magnetostriction Oscillator

Magnetostriction is a property of ferromagnetic materials that causes them to change their shape when subjected to magnetic field. The effect is first identified in 1842 by James Joule. e.g. iron, nickel etc.

According to principle if ferromagnetic material in the form of a bar, is subjected to a alternating magnetic field produced due to oscillatory circuit. Due to longitudinal frequency of crystal, then ultrasonic waves are produced with high amplitude.

Construction: the circuit diagram is shown in fig. A slice of quartz crystal is placed between the metal plates 'A' and 'B' so as to form a parallel plate capacitor with the crystal as dielectric. This is coupled to the electronic oscillator through the primary coil L_3 of the transformers. Coils L_1 and L_2 are taken as the primary of transformer. The collector coil L_2 is inductively coupled to base coil L_1. The coil C_1 and variable capacitor 'C' forms the tank circuit of the oscillator.

Working: When the circuit is ON, the oscillator produces high frequency oscillations. Due to transformer action, an emf is induced in the coil L_3. The capacitor L_1 is adjusted so that oscillations produced in resonance with natural frequency of crystal. Now crystal vibrates with larger amplitude, ultrasonic waves are produced.

Then the frequency of vibrating crystal is given by,

$$f = \frac{P}{2l}\sqrt{\frac{y}{\rho}}$$

Where, l = length of the rod;

y = Youg's modulus or elasticity of crystal;

ρ = density of material of crystal;

adjusting the capacitor so that frequency of oscillator circuit is equal to the natural frequency of vibrating rod.

The condition of resonance is –

$$f = \frac{1}{2l}\sqrt{\frac{y}{\rho}} \quad \text{or} \quad f = \frac{1}{2\pi\sqrt{L_1 C_1}}$$

Where, l = length of the rod;

y = Youg's modulus or elasticity of crystal;

ρ = density of material of crystal;

The oscillator can produce frequencies up to 3MHz only.

***Advantages*:**

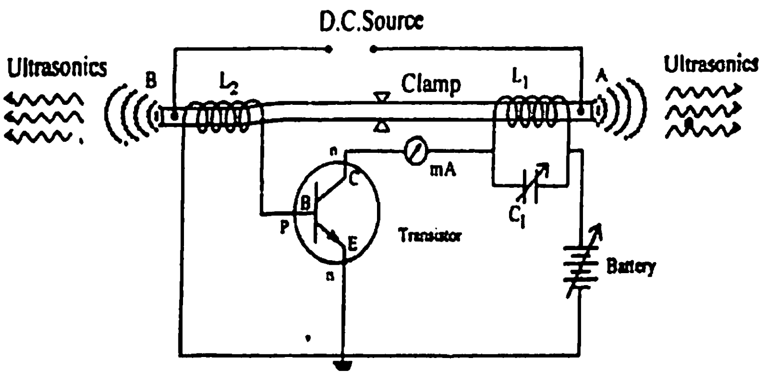
- The construction cost is low.
- They are capable for producing with fairly good efficiency.

4.10 Detection of ultrasonic waves

They are many methods to detect the ultrasonic waves when passing through medium (1) Kundt's tube method (2) Sensitive flame method (3) Thermal detectors (4) Quartz crystal method.

[1] Kundt's tube method: It is invented in 1866 by German Physicist Augus Kundt. The tube is a transparent horizontal pipe which contains a small amount of fine contraction and expansion of the bar, longitudinal compression waves are produced in the medium surrounding the bar.

Principle: It is bases on the principle of magnetostriction effect.



Construction: The circuit diagram is shown in fig. A Nickel rod is clamped in the middle between two knife edges. A coil L_1 is wound on the right hand portion of the rod. 'C' is a variable capacitor, 'L' and 'C_1' forms the resonant circuit of collector tuned oscillator. Coil L_2 wound on the LHS of the rod is connected in the base circuit. The coil L_2 is used as a feedback loop.

Working: When the circuit is ON, the L_1 and C_1 forms the resonant circuit, the rod stars vibrating due to magnetostriction effect. The longitudinal expansion and contraction of rod produces the emf in the coil L_2. This induced emf is applied to the base of transistor. By nodes is equal to half the wavelength. If we know frequency (f) then velocity of ultrasonic wave can be calculated.

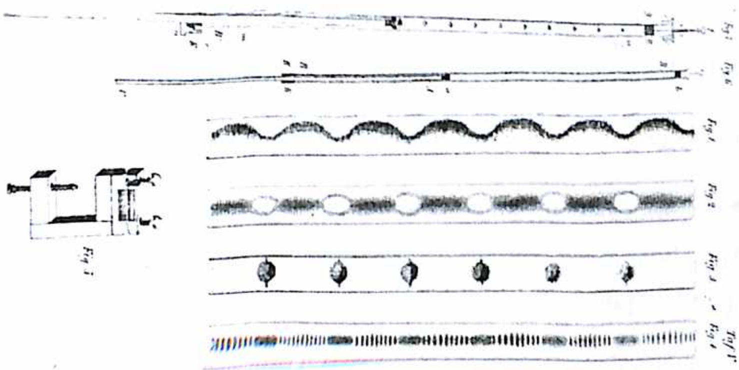
[3] **Thermal detectors:** When ultrasonic waves pass through a medium, then alternating compressions and rarefactions are formed. In this method, a fine platinum wire is used; this wire is moved through the medium. At position of nodes, due to compressions and rarefactions, changes in temperature takes place, so resistance of wire changes with respect to time. It is found at the antinodes, the temperature remains constant and resistance of wire remains constant. This can be detected by calendar and Garrifith's bridge arrangement.

[4] **Quartz crystal method:**This method is based on the principle of piezo electric effect. When two opposite faces of quartz crystal is exposed to ultrasonic wave, then the opposite charges are formed on the opposite pair of faces. It can be detected by the electronic circuit.

4.11 Acoustic grating:

Principle: When ultrasonic waves are passed through a liquid, the density of the liquid varies layer by layer due to variation in pressure and hence liquid will act as a diffraction grating, so called acoustic grating. Under this condition, when monochromatic source of light is passed through a acoustical grating, the light gets diffracted. Then by using the condition for diffraction, the velocity of ultrasonic waves can be determined.

powder such as Lycopodium or talk or cork. At one end of tube the source of sound and other end is blocked by a movable piston to adjust the length of tube. When the tube is at resonance, large sound is produced and stationary wave is produced in the tube. At the nodes lycopodium powder collects in the form of heaps. The average distance between two adjacent heaps is equal to the half the wavelength. This method can't be used if λ is very small i.e. less than millimeters. In case of liquid medium, instead of powder, coke is used to detect the position of nodes.



[2] Sensitive flame method: A narrow sensitive flame is moved along the medium. At the position of the antinodes, the flame is steady. At the position of the node, the flame flickers because there is change in pressure. In this way positions of nodes and antinodes can be found out in a medium. The average distance between two adjacent diffracted beam of light is viewed through the telescope. The diffraction pattern consists of a central maximum and principal maxima on either side. The direction of principal maxima are given by the expression,

$$d\sin\theta_n = n\lambda \text{ .... } (1)$$

Here, $n = 1, 2, 3, \ldots$ etc. are the order

θ_n = angle of diffraction for n^{th} order

λ = wavelength of source

d = distance between two adjacent nodes or antinodes

By knowing n, θ_n and λ the value of 'd' can be calculated,

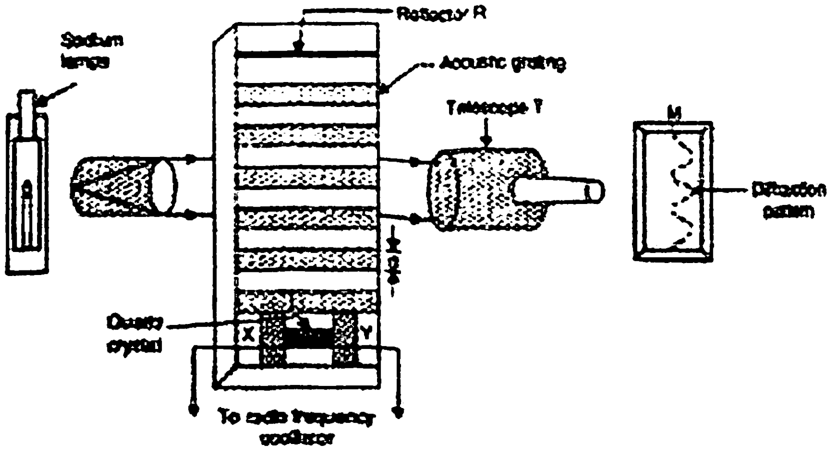
$$\therefore d = \lambda / 2 \qquad \Rightarrow \lambda = 2d \text{ .... } (2)$$

If 'N' is the resonant frequency of piezo electric oscillator and 'V' velocity of ultrasonic wave is given by–

$V = N\lambda$ ...... put this value in equation (2)

$$\therefore V = 2 \times N \times d \text{ .... } (3)$$

Thus velocity of ultrasonic waves through liquids and gases can be measured at various temperatures by using above equation.



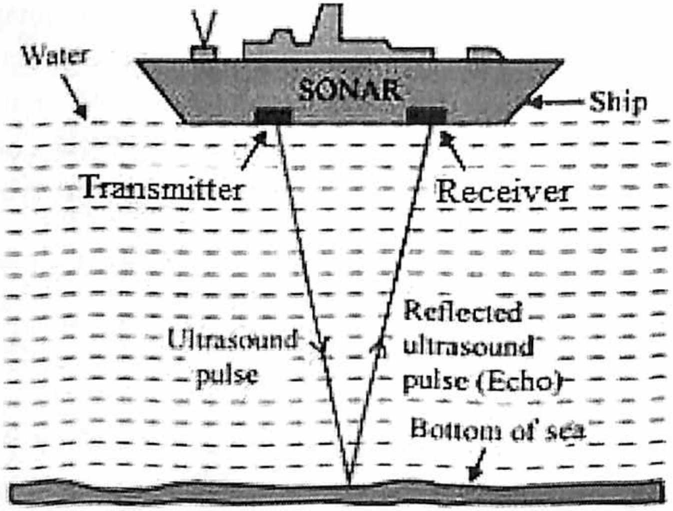
Construction & Working:

The liquid is taken in a glass cell. The piezo electric crystal (Quartz) is fixed at one side of the wall inside the cell and ultrasonic waves are generated. The waves travelling from the crystal get reflected by the reflector placed at the opposite wall. The reflected wave gets superimposed with the incident waves producing longitudinal standing wave pattern called acoustic grating.

This arrangement is suitable for the measurement of wavelength and velocity of ultrasonic waves in a medium as shown in figure.

The acoustic grating is mounted on the prism table of spectrometer. A parallel beam of sodium light from the colimeter is normally incident on the acoustic grating. The

- The instrument calibrated to show the depth of sea is called *fathometer* or *echo-meter*.
- A fish-finder or sounder is an instrument used to locate fish underwater sound energy.



- Fathom is a unit of length in the imperial measuring mechanically the depth of water beneath a ship.

[2] **Medical applications:**

(a) **Neuralgic pain:** Ultrasonic waves are useful for relieving neuralgic pain. The affected portion of the body is exposed to ultrasonic waves; it produces a soothing massage action and relieves pain.

(b) **Broken teeth:** Ultrasonic waves are used by dentists for the proper extraction of broken teeth.

4.12 Applications of Ultrasonic Waves

[1] Measurement of depth of the sea:

The piezo electric quartz oscillator is used for this purpose. Quartz crystal itself acts as a transmitter and receiver of the ultrasonic waves. The ultrasonic waves are transmitted towards bed of the sea and then these waves are reflected back from the bed and echo is detected by the quartz crystal itself. The time interval between the emitted signal and the reflected echo is determined with the help of oscillograph.

Then V = velocity of sound through sea water

Time = t

Then depth of sea (h) is given by,

Velocity = depth/ time

$V = 2h/t$

$\therefore h = vt/2$

This information is typically used for navigation purposes.

- This method is also used to detect the presence and depth of submarines, rocks etc. from surface of water.

can also be destroyed by applying the ultrasonic waves.

[3] Detection of Cracks of Flaws in Metals:

If there is any hidden crack in metal structure it acts as a good reflector for ultrasonic waves and the cracks can be detected.

[4] Heating effects:

When ultrasonic beam is passed through substance, it gets heated.

[5] Mechanical effects:

Ultrasonic drills are used to bore holes in steel and other metals or their alloys. It is used to cleaning cloths specially woolen and silken fibers.

- They are used for dispersing fog at airports.
- Cleaning the objects such as parts of watches.

[6] Formation of alloys:

Alloys of uniform composition are obtained by applying an ultrasonic beam.

[7] Chemical effect:

(a) Ultrasonic waves act like catalytic agents and accelerate chemical reactions.

(c) **Bloodless surgery:** Ultrasonic waves are focused on a sharp instrument and the tissues are destroyed without any loss of blood. Highly focused is used in neurosurgery, it is only method for producing trackless damage deep in the brain.

(d) **Sonography (Ultrasonography):** It is imaging techniques used to visualize body structures including muscles, joints, vessels and internal organs. It is effective for imaging soft tissues of the body used to examine a baby in pregnant women and the brain and hips in infants.

It is used to locate the exact position of an eye tumor and its removal and give normal vision to the patient.

(e) Used in the treatment of mental patients, such as electric shock, psychotherapy.

(f) Sterilization: Ultrasonic waves destroy the organism, bacteria and used in the sterilization of water and milk. Extra red blood corpuscles

- Prof. Harridge of London experiments on bats how they can avoid obstacles while flying in complete darkness.
- During flight they produce ultrasonic waves in a series of pulses and receives reflected wave from object and interprets the time between transmitted and received signals and distance is estimated, bats are used 50,000Hz frequency.

4.13 Exercises

Que. 1 State the field of acoustics.

Que. 2 What is Reverberation and on which factors does it depend?

Que. 3 Define Reverberation time and on which factors does it depend?

Que. 4 Derive Sabine's reverberation time formula?

Que. 5 What is absorption coefficient?

Que. 6 What is acoustic measurement and obtains the equation of acoustic Intensity and Pressure Intensity level?

Que. 7 State the conditions of good acoustical design of auditorium.

Que. 8 What is ultrasonics? Explain in brief.

(b) Iodine is liberated from a solution of potassium iodide and sulphur from hydrogen sulphide.

(c) As water and oil are immiscible. An emulsion of water and oil is obtained when mixture is exposed to ultrasonic waves. Also water and mercury emulsion can be prepared.

[8] Soldering:

Aluminum can't be soldered by ordinary soldering method. To solder aluminum, ultrasonic waves are used in addition to electrical soldering.

[9] Enemy of lower life:

When some lower life animals like rat, frogs, fish etc. are exposed to ultrasonic waves they become lame.

[10] Ultrasonic waves are used for determination of may parameters like specific heat, compressibility, absorption coefficients, concentration, atomicity of molecules, chemical structure, elastic symmetry of crystal etc.

[11] Ultrasonic in Nature:

Ultrasonic waves can be responded by the dog to call the dog. A whistle (ultrasonic wave) can be used. So it will not annoy the neighbors with any noise. In America ultrasonic's are used to scare away the birds.

MCQ - EXERCISES

1. What is the change of phase if a wave is reflected from a denser medium?

   (a) 3π (b) O

   (c) π (d) 2π

2. The differential equation of a wave motion is ....

   (a) $\frac{d^2y}{dt^2} = V^2 . \frac{d^2y}{dx^2}$ (b) $\frac{d^2y}{dx^2} = V^2 . \frac{d^2y}{dt^2}$

   (c) $\frac{d^2y}{dt^2} = \frac{d^2y}{dx^2}$ (d) None

3. A wave of the form A cos(wt + π) is added to a string carrying a wave of the form A cos(wt). What is the equation of resulting wave?

   (a) O (b) 2A coswt

   (c) A cos(wt + π/2) (d) None

4. The elasticity of material medium is ......

   (a) stress × strain (b) $\frac{stress}{strain}$

   (c) stress^2 × strain (d) None

5. The velocity of the wave through the medium of elasticity (E) and density (ρ) is ....

   (a) $V = \sqrt{E \cdot \rho}$ (b) $V = \sqrt{E^2 \cdot \rho^2}$

   (c) $V = \sqrt{\frac{\rho}{E}}$ (d) $V = \sqrt{\frac{E}{\rho}}$

6. The total energy of progressive wave is ......

   (a) T.E. = K.E. + P.E. (b) T.E. = K.E. – P.E.

   (c) T.E. = K.E./P.E. (d) T.E. = 0

Que. 9 What is peizo electric effect? Describe construction, working of piezo electric oscillator.

Que. 10 What is magnetostriction effect? Describe construction, working of magnetostriction oscillator.

Que. 11 Explain how the velocity of ultrasonic wave is determined by using acoustic grating.

Que. 12 Explain the construction and working of acoustic grating?

Que. 13 State any five applications of ultrasonic waves.

Que. 14 Describe briefly the applications of ultrasonic waves.

14. The frequency is measured by ......

(a) second (b) cycles/second

(c) Hertz (d) Both (b) and (c)

15. One Mega Hertz is the ......

(a) 10^2 Hz (b) 10^3 Hz

(c) 10^6 Hz (d) 10^9 Hz

16. The frequency at which a system tends to oscillate in the absence of any driving or damping force is called.....

(a) Fundamental frequency (b) Forced frequency

(c) Natural frequency (d) None

17. 1 GHz = ...Hz

(a) 10^9 (b) 10^6

(c) 10^3 (d) 10^2

18. According to research, the average natural frequency of a human body is about ......

(a) 1 Hz (b) 100 Hz

(c) 7.5 Hz (d) 2.5 Hz

19. In stationary waves, the strain is maximum at ......

(a) Antinodes (b) Nodes

(c) Both (a) and (b)(d) None

20. The antinodes are those positions where the strain is...

(a) Zero (b) Minimum

(c) Maximum (d) None

21. What will be the wave velocity, if radar gives 54 waves/min and wavelength 10m?

(a) 4 m/s (b) 6 m/s

(c) 9 m/s (d) 5 m/s

7. The equation of total energy of progressive wave through medium having density (ρ), amplitude (a) and frequency (η) is ......

(a) $T.E. = 2\pi\rho\eta$ (b) $T.E. = 4\pi^2\rho^2\eta^2$
(c) $T.E. = 2\pi^2\rho\eta^2$ (d) Zero

8. When a string is stretched and is plucked at centre and released then it vibrates ...... to the length of string.

(a) parallel (b) perpendicular
(c) Both (a) & (b) (d) None

9. In the above situation the resultant component of vibration is ......

(a) vertical (b) horizontal
(c) Both (a) & (b) (d) None

10. The rate of change of displacement of the particle give the ......

(a) force (b) velocity
(c) acceleration (d) momentum

11. The square rate of change of displacement of the particle gives the ......

(a) momentum force (b) force
(c) velocity (d) acceleration

12. The equation of centripetal force is ......

(a) $F = mv/R$ (b) $F = mv^2$
(c) $F = ma$ (d) $F = mv^2/R$

13. The number of cycles/vibrations completed by the particle in one second is called ....

(a) period (b) frequency
(c) velocity (d) None

28. The rate of fall of current in damped SHM electrical circuit is ......

(a) dQ/dt (b) dI/dt
(c) dv/dt (d) None

29. In damped vibration if $\frac{R^2}{4L^2} = \frac{1}{LC}$ then discharge is......

(a) Critically damped (b) Oscillatory
(c) Normal (d) None

30. In damped vibration, if $\frac{R^2}{4L^2} < \frac{1}{LC}$ then discharge is......

(a) Critically damped (b) Oscillatory
(c) Normal (d) None

31. The higher frequency of sound than 20,000Hz are ....

(a) Infrasonic (b) Ultrasonic
(c) Both (a) & (b) (d) None

32. The production of ultrasonic wave by peizo-electric oscillator is based on the principle......

(a) peizo electric effect (b) photo electric effect
(c) Crompton effect (d) none

33. By using piezo electric oscillator ultrasonic frequency as high as ......Hz can be obtained

(a) 5×10^2 (b) 5×10^3
(c) 5×10^8 (d) 5×10^{20}

34. In Acoustic grating arrangement, velocity of ultrasonic wave is given by......

(a) v = Nd (b) Nd/2
(c) N^2d (d) 2Nd

22. In a transverse wave angle between particle velocity and wave velocity is ......

(a) Zero (b) $\pi/4$

(c) $\pi/2$ (d) π

23. The stationary waves of frequency 300 Hz are formed in a medium in which the velocity of sound is 1200m/s. The distance between a node and neighboring antinode is...

(a) 1m (b) 2m

(c) 3m (d) 4m

24. When stationary wave is formed, then is frequency is...

(a) Same as that of individual waves

(b) Twice that of individual waves

(c) Half that of individual waves

(d) Zero

25. Regarding as open organ pipe, which statement is correct?

(a) Both ends are pressure antinodes

(b) Both ends are displacement nodes

(c) Both ends are pressure nodes

(d) None

26. When body vibrates with free vibrations then its frequency is called

(a) Fundamental frequency (b) Resonance

(c) Forced (d) Natural frequency

27. When body vibrates having frictional forces and energy is lost and amplitude die out, vibrations are called ....

(a) free damped vibration (b) undamped vibration

(c) forced vibration (d) None

42. The relation between particle velocity (U), wave velocity (v) is......

(a) $V = U \times \frac{dy}{dx}$ (b) $U = V \times \frac{dy}{dx}$

(c) $\frac{dy}{dx} = U \cdot V$ (d) none

43. Velocity of transverse wave along a string is......

(a) $V = \sqrt{T \cdot m}$ (b) $V = \sqrt{\frac{m}{T}}$

(c) $V = \sqrt{\frac{T}{m}}$ (d) none

44. In stationary waves, when strain is zero then positions corresponds to ......

(a) Antinodes (b) Nodes

(c) Both (a) & (b) (d) None

45. In stationary waves, the distance between successive Nodes is ......

(a) λ (b) $\lambda/2$

(c) 2λ (d) $\lambda/3$

46. In one periodic time (T), particle passes ...... times through mean postion.

(a) 1 (b) 2

(c) 3 (d) 4

47. A node point in a stationary wave, the amplitude of the particle is...

(a) Maximum (b) Normal

(c) Zero (d) None

35. Production of ultrasonic waves by magnetostriction oscillator is based on principle of……

(a) magnetostriction effect (b) peizo electric effect
(c) photo electric effect (d) none

36. In acoustic measurement, 10gimic scale used for measurement is called ……

(a) Kelvin's scale (b) degree scale
(c) decibel scale (d) none

37. 10 decibels (db)

(a) 10 bel (b)100 bel
(c) 1 bel (d) 0.1 bel

38. The reverberation in auditorium can be improved by using the surfaces with …… absorption coefficient.

(a) high (b) low
(c) zero (d) none

39. For improvement of reverberation, the curved walls and corners bounded by two walls should be avoided to avoid.

(a) dead spaces (b) bad look
(c) corners (d) none

40. When lower life animals are exposed by ultrasonic's the effect on their body is……

(a) good (b) helps to increase life
(c) dangerous (d) nothing

41. The relation between wave velocity (v), frequency (η) and wavelength (λ) is……

(a) $v = \eta \lambda$ (b) $v = \eta/\lambda$
(c) $v = \eta^2\lambda$ (d) $v = \eta\lambda^2$

55. The frequency of oscillation is given by –

(a) $f = 1/2\pi \sqrt{\frac{K}{m}}$ (b) $f = 2\pi \sqrt{\frac{K}{m}}$

(c) $f = 2\pi \sqrt{\frac{m}{K}}$ (d) none

56. In free damped vibrations, amplitude of swing decreases and finally become......

(a) maximum (b) very high

(c) zero (d) can't say

57. In LCR circuit the current is given by......

(a) $I = \frac{dQ}{dt}$ (b) $I = \frac{d^2Q}{dt^2}$

(c) $I = Q \cdot t$ (d) None

58. In damped vibration, the discharge is aperiodic and critically damped if

(a) $\frac{R^2}{4L^2} > \frac{1}{LC}$

(b) $\frac{R^2}{4L^2} < \frac{1}{LC}$

(c) $\frac{R^2}{4L^2} = \frac{1}{LC}$

(d) none

59. In damped vibration, when $\frac{R^2}{4L^2} < \frac{1}{LC}$ then discharge is......

(a) Non oscillatory (b) Oscillatory

(c) Aperiodic (d) Normal

60. In stationary wave, the rate of transfer of energy is

(a) maximum (b) double

(c) minimum (d) zero

48. At Antinode points, the amplitude of the particle is...

(a) Maximum (b) Normal

(c) Zero (d) None

49. At Nodes, the change in pressure is......

(a) Zero (b) Minimum

(c) Maximum (d) None

50. In case of stationary wave, the total energy at any instant is ...... that of progressive wave.

(a) double (b) single

(c) equal (d) triple

51. When body vibrating freely has no resistance offered to its motion, its amplitude remains......

(a) Zero (b) Maximum

(c) Minimum (d) Constant

52. In S.H.M. the K.E. is given by ....

(a) $\frac{1}{2} \times m \times v^2$ (b) $2mv^2$

(c) $\frac{mv^2}{2}$ (d) $\frac{m^2v^2}{2}$

53. The differential equation of undamped free vibration

(a) $\frac{dy}{dt} + \omega y = 0$ (b) $\frac{d^2y}{dt^2} + \omega^2 y = 0$

(c) $\frac{d^2y}{dt^2} + \omega^2 y^2 = 0$ (d) None

54. In case of S.H.M. the value of ω^2 (angular velocity) is

(a) K·m (b) m/K

(c) K^2/m (d) K/m

68. Acoustics of an auditorium can be improved by –
   (a) Hanging heavy curtains
   (b) Having picture and maps
   (c) Having few open widows
   (d) All above

69. Persistence of sound even after the source has stopped is known as
   (a) Resonance (b) Absorption coefficient
   (c) Ultrasonic (d) Reverberation

70. In stationary waves, the distance between successive node is 1.2cm then the distance between successive antinode is......
   (a) 1.2 cm (b) 0.8cm
   (c) 1.0cm (d) 0.6cm

71. The acoustics branch of physics is deals with the......
   (a) Generation of sound (b) Reception
   (c) Propagation of sound (d) All above

72. The limits of human audibility is.......
   (a) 20 Hz to 20 KHz (b) Lower 20 Hz
   (c) Greater 20 KHz (d) All above

73. The sound waves of ultrasonic have the frequency...
   (a) less than 20,000 Hz (b) greater than 20,000 Hz
   (c) 20 Hz to 20 KHz (d) None

74. Magnetostriction oscillator produces the ...... waves.
   (a) Micro (b) Ultrasonic
   (c) Cosmic (d) None

61. When some external periodic force is constantly applied on a body, it continues to oscillate under its influence; such vibrations are called......

(a) free vibrations
(b) forced vibrations
(c) resonance
(d) normal vibrations

62. Resonance is the special case of......

(a) free damped vibration
(b) free undamped vibration
(c) forced vibration
(d) none

63. The existence of damping can......

(a) decreases in amplitude
(b) increases in amplitude
(c) maintain constant amplitude
(d) none of above

64. At resonance, amplitude of oscillation is......

(a) zero
(b) minimum
(c) In between zero and maximum
(d) maximum

65. Aperiodic motion is also called as.......

(a) Dead beat
(b) Critically damped motion
(c) Oscillatory motion
(d) None of above

66. In peizo electric oscillator, the fundamental frequency is.....

(a) $F = \frac{P}{2l}\sqrt{\frac{E}{\rho}}$
(b) $F = \frac{P}{l}\sqrt{\frac{E}{\rho}}$
(c) $F = \frac{2P}{l}\sqrt{\frac{E}{\rho}}$
(d) none

67. Piezo electric property is more pronounced found in crystal of

(a) Quartz
(b) Tourmaline
(c) Rochellel salt
(d) In all above

82. In a winding (spring) watch, the energy is stored in the form of

(a) kinetic energy
(b) potential energy
(c) electrical energy
(d) none of above

83. The ...... vibrations taking place in the diaphargon of a microphone

(a) free
(b) damped
(c) forced
(d) electrical

84. In case of sustained forced oscillations the amplitude.......

(a) decreases
(b) increases
(c) remains constant
(d) can't say

85. Amplitude of vibrations remain constant in case of

(i) free vibrations
(ii) damped vibrations
(iii) maintained vibrations
(iv) forced vibrations

(a) (i), (iii), (iv)
(b) (ii), (iii)
(c) (i), (ii), (iii)
(d) (ii), (iv)

86. When sound wave travels from air to water .... remains constant

(a) wavelength
(b) velocity
(c) frequency
(d) intensity

87. The energy in superposition of waves is.......

(a) lost
(b) increase
(c) remains same
(d) none

75. The ultrasonic waves production method was developed by a ......

(a) Pierce (b) Newton
(c) Langevin (d) Eienstein

76. Velocity of ultrasonic waves can be measured by using......

(a) acoustic grating (b) sonometer
(c) rheostat (d) screw gauge

77. The time that the sound takes to fall in intensity by 60 decibels or to one millionth (10^{-6}) to its original intensity after the source is stopped called......

(a) period of oscillation (b) period of pendulum
(c) period of reverberation (d) None

78. The equation of period or reverberation is

(a) $T = \frac{0.05V}{\Sigma a_1 s_1}$ (b) $T = \frac{0.5V}{\Sigma a_1 s_1}$
(c) $T = \frac{0.05V^2}{\Sigma a_1 s_1}$ (d) None

79. Absorption coefficient can be determined by the method

(a) simple pendulum (b) Reverberation chamber
(c) sonometer (d) acoustic grating

80. Ultrasonic waves are the type of

(a) light waves (b) Radiations
(c) sound waves (d) electromagnetic waves

81. The phase of particle in S.H.M is $\pi/2$ then......

(a) velocity (b) acceleration
(c) restoring force (d) displacement

(a) None magnetic resonance
(b) Nuclear memory reader
(c) Nuclear magnetic red
(d) Nuclear magnetic resonance

97. In ultra sonography instrument the frequency is used
(a) Audio (b) Infrasonic
(c) Microwave (d) Ultrasonic

98. For a good auditorium it is necessary to keep the reverberation time
(a) Very high (b) Small
(c) Zero (d) can't say

99. Architectural acoustic has been studied by the...
(a) Prof. W.C. Sabine (b) Prof. Narlikar
(c) Kirchhoff (d) Newtone

100. Echelon effect in cause of sound is related to
(a) reflection (b) polarization
(c) echo (d) photo electric effect

Answer Key:

1 (c)	11 (d)	21 (c)	31 (b)	41 (a)	51 (d)	61 (b)	71 (d)	81 (d)	91 (c)
2 (a)	12 (d)	22 (c)	32 (a)	42 (b)	52 (a)	62 (c)	72 (a)	82 (b)	92 (d)
3 (a)	13 (b)	23 (a)	33 (c)	43 (c)	53 (b)	63 (a)	73 (b)	83 (c)	93 (a)
4 (b)	14 (d)	24 (a)	34 (d)	44 (a)	54 (d)	64 (d)	74 (b)	84 (c)	94 (b)
5 (d)	15 (c)	25 (c)	35 (a)	45 (b)	55 (b)	65 (b)	75 (c)	85 (a)	95 (a)
6 (a)	16 (a)	26 (d)	36 (c)	46 (b)	56 (c)	66 (a)	76 (a)	86 (c)	96 (d)
7 (c)	17 (c)	27 (a)	37 (c)	47 (c)	57 (a)	67 (d)	77 (c)	87 (c)	97 (d)
8 (b)	18 (c)	28 (b)	38 (a)	48 (a)	58 (c)	68 (d)	78 (a)	88 (a)	98 (b)
9 (a)	19 (b)	29 (a)	39 (a)	49 (c)	59 (b)	69 (d)	79 (b)	89 (a)	99 (a)
10 (b)	20 (a)	30 (b)	40 (c)	50 (a)	60 (d)	70 (c)	80 (c)	90 (a)	100 (C)

88. With the increase of temperature, the frequency of the organ pipe

(a) increase (b) decreases

(c) remains constant (d) can't say

89. Node points of stationary wave is separated by a distance........

(a) $\lambda/2$ (b) λ

(c) $3\lambda/2$ (d) 2λ

90. Antinode points of stationary wave is separated by a distance

(a) $\lambda/2$ (b) λ

(c) $3\lambda/2$ (d) 2λ

91. Nodes are those points of medium when stationary wave is travelling at which the strain is.......

(a) zero (b) minimum

(c) maximum (d) variable

92. Free vibrations are depends upon

(a) time (b) direction

(c) place (d) dimension

93. In general practically .... Are does not exists

(a) free vibrations (b) forced vibrations

(c) resonance (d) none

94. The angular velocity (ω) is given by.......

(a) $2\pi T$ (b) $2\pi f$

(c) $2\pi T^2$ (d) $2\pi f^2$

95. The amplitude of vibration of a body is equal to the ...... between the natural frequency and applied force frequency

(a) Difference (b) Addition

(c) Multiplication (d) None

96. NMR can be read as

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