

BAHIRJI SMARAK MAHAVIDYALAYA, BASMATH

Dist. Hingoli, Maharashtra

Department of Physics

STUDY NOTES

Fundamentals of Physics – II

SPHYCT1151

Optical Instruments · Electromagnetism
Electrical Circuit Analysis · Kinetic Theory of Matter

Course Code	SPHYCT1151
Course Title	Fundamentals of Physics – II
Programme	B.Sc. First Year (Physics) · 4-Year UG Programme, NEP 2020
University	Swami Ramanand Teertha Marathwada University (S.R.T.M.U.), Nanded
Credits / Hours	2 Credits · 30 Theory Hours · No Practical
Units	I Optical Instruments · II Electromagnetism · III Electrical Circuit Analysis · IV Kinetic Theory of Matter

For B.Sc. First Year Physics students

Concise theory · derivations · diagrams · solved numericals · question bank

By

Dr. Narsingh

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ABOUT · Overview hours

About this course and how to use these notes

Course at a glance

SPHYCT1151: Fundamentals of Physics – II is a Major (Theory) course of the four-year UG programme under the Faculty of Science and Technology, S.R.T.M.U., Nanded. It introduces four building blocks of physics that every science student needs: how optical instruments extend human vision, how changing magnetic flux produces electricity, how electrical networks are analysed systematically, and how the behaviour of gases follows from the motion of molecules.

Item	Details
Course code and title	SPHYCT1151 – Fundamentals of Physics-II
Teaching scheme	Theory 02 hours per week; Practical nil
Credits assigned	Theory 02; Practical nil; Total 02
Total teaching hours	30 (Unit I: 07, Unit II: 08, Unit III: 08, Unit IV: 07)
Assessment	Total 50 marks (theory only): continuous assessment through Test I and Test II (average taken) plus end-semester assessment. Check the official syllabus for the exact split.

Prerequisites, objectives and outcomes

Prerequisites. Basic knowledge of physics and the properties of materials; familiarity with the terminology of physical sciences and its everyday applications; and a working knowledge of optics, electromagnetism, electrical circuit analysis and the kinetic theory of matter at higher-secondary level.

Course objectives

- To introduce the fundamentals of physics in brief, covering optics, electromagnetic properties and the kinetic theory of matter.
- To help students understand the laws of light interaction and of electronic motion, and their applications to various systems.

Course outcomes

- Students will understand the fundamental nature of physics.
- Students will be able to handle different types of problems and will be ready for advanced courses in physics.

Syllabus and hours

Unit	Topics	Hours
I. Optical Instruments	Objective and eyepieces; Huygens and Ramsden eyepieces (cardinal points omitted) and their comparison; compound microscope; telescope; reflecting telescope; constant deviation spectrometer	07
II. Electromagnetism	Faraday's law and Lenz's law; self-induction and self-inductance of a coil and of a solenoid; mutual induction and mutual inductance of a pair of co-axial coils and of co-axial solenoids	08
III. Electrical Circuit Analysis	R, L, C elements; Ohm's law; Kirchhoff's laws; series and parallel resistors; superposition, Thevenin, Norton and maximum power transfer theorems	08
IV. Kinetic Theory of Matter	Three states of matter; ideal gas and kinetic model; pressure of a gas; r.m.s. speed; deduction of Boyle's law; gas equation; other gas laws	07

How to use these notes

Each unit follows the syllabus order, so you can tick off topics as the class proceeds. Boxes marked **Key points** hold the statements and formulae you should be able to write from memory; green **Example** blocks show a complete numerical solution that you can use as a template. The last section collects an all-units formula sheet, short and long questions, multiple-choice questions and numerical practice problems with answers. When you write a derivation in an examination, always draw the neat labelled diagram first: most marks are lost by leaving it out.

Examination tips

- Learn each derivation as a short chain of steps (set-up, diagram, key equation, result); examiners reward the sequence of reasoning.
- Always state the **units** of the final answer and quote the formula before substituting numbers.
- For circuit problems, label currents and polarities on a clean diagram before writing any equation.
- Revise the comparison tables (eyepieces, self and mutual induction, Thevenin and Norton) just before the test; they are favourite short-answer questions.

Constants and conventions used

Quantity	Symbol and value
Permeability of free space	$\mu_0 = 4\pi \times 10^{-7} \text{ H m}^{-1}$
Boltzmann constant	$k = 1.38 \times 10^{-23} \text{ J K}^{-1}$
Universal gas constant	$R = 8.314 \text{ J mol}^{-1} \text{ K}^{-1}$

Quantity	Symbol and value
Avogadro number	$N_A = 6.022 \times 10^{23} \text{ mol}^{-1}$
Least distance of distinct vision	$D = 25 \text{ cm}$

UNIT I · 07 hours

Optical Instruments

1.1 Introduction to optical instruments

Optical instruments are devices that use lenses, mirrors and prisms to form images that help the eye see objects that are too small, too far away, or too faint, and to analyse light itself. The **microscope** magnifies tiny nearby objects, the **telescope** brings distant objects closer, and the **spectrometer** separates light into its wavelengths. All of them are built from two basic parts: an **objective**, which collects light from the object and forms a first (real) image, and an **eyepiece**, which magnifies that image so the eye can examine it.

A single thin lens suffers from several defects (aberrations). The two that matter for eyepieces are **spherical aberration**, in which rays through the edge of the lens focus at a different point from rays near the axis, and **chromatic aberration**, in which different colours focus at different points because the refractive index depends on wavelength. Designers reduce both by replacing one lens with a combination of two lenses of the same material, separated by a suitable distance.

1.2 The objective and the eyepiece

The **objective** is the lens (or mirror) nearest the object. In a microscope it has a very short focal length and a large aperture so that it can form a magnified real image of a small object placed just beyond its focus. In a telescope it has a long focal length and large aperture so that it gathers as much light as possible from a distant object.

The **eyepiece** (ocular) is the lens system nearest the eye. It works as a simple magnifier on the image formed by the objective. A good eyepiece should give a wide, flat field of view, a bright image, small aberrations and a comfortable position for the eye (eye relief). A single lens cannot do this, so practical eyepieces use two plano-convex lenses: the **field lens**, facing the objective, and the **eye lens**, facing the observer. The field lens bends the cone of rays inwards so that more of the objective's image enters the eye lens, which increases the field of view.

Equivalent focal length of two thin lenses

Two thin lenses of focal lengths f_1 and f_2 separated by distance d (in air) behave as a single system of equivalent focal length F given by

$$1/F = 1/f_1 + 1/f_2 - d/(f_1 f_2)$$

Two useful conditions for lenses of the same material follow from the theory of aberrations:

- **Achromatism (for the equivalent lens):** the combination is free from chromatic aberration when $d = (f_1 + f_2)/2$.
- **Minimum spherical aberration:** for the same pair, spherical aberration is least when the deviation is shared equally, which is closely met when $f_1 - f_2 = d$ with the more curved faces towards the light (Huygens eyepiece).

1.3 Huygens eyepiece

Construction. Two plano-convex lenses of the same glass are mounted in a brass tube with their **convex faces towards the incident light**. The field lens L_1 has focal length $3f$ and the eye lens L_2 has focal length f , and they are separated by $d = 2f$. A circular **field stop** (diaphragm) is placed midway between them, at the position where the intermediate image is formed.

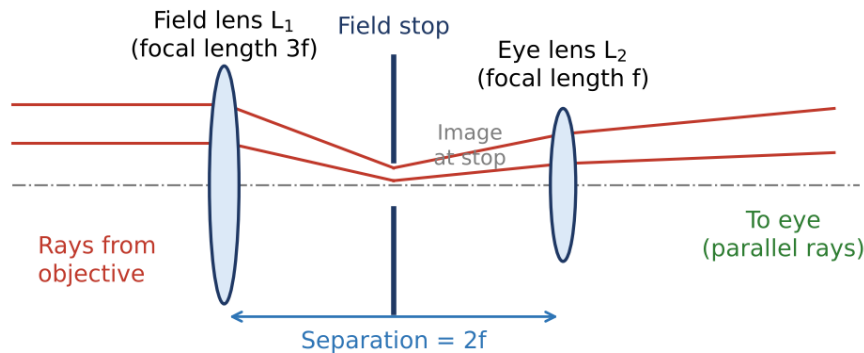


Figure 1.1: Huygens eyepiece (a negative eyepiece with the image formed between the lenses)

Working. The objective alone would form its image at a distance $3f/2$ beyond the field lens. The field lens, however, intercepts the converging rays first and shifts the image to a distance f from itself, i.e. exactly at the focal plane of the eye lens (since $2f - f = f$). The image formed between the two lenses is therefore **virtual for the field lens and real in front of the eye lens**: it is a **negative eyepiece**. The eye lens then acts as a simple magnifier and sends parallel rays to the relaxed eye, giving a final image at infinity.

Achromatism. Since $d = 2f$ and $(f_1 + f_2)/2 = (3f + f)/2 = 2f$, the condition $d = (f_1 + f_2)/2$ is satisfied. The eyepiece is therefore free from lateral chromatic aberration, and by using the equal-deviation arrangement the spherical aberration is also very small.

Equivalent focal length. Using the formula above,

$$1/F = 1/(3f) + 1/f - 2f/(3f \cdot f) = 2/(3f) \Rightarrow F = 3f/2$$

Limitations. Because the image is formed **between** the lenses, cross-wires or a scale placed at the field stop are viewed through the eye lens alone, whose own aberrations are not corrected for that image. Hence a Huygens eyepiece cannot be used for measurements; it is used only for **observation** (e.g. in ordinary microscopes and telescopes).

1.4 Ramsden eyepiece

Construction. Two plano-convex lenses of the **same focal length f** are mounted with their **convex faces towards each other**. The separation is made $d = 2f/3$, so that the eye lens lies a little nearer than the focal length of the field lens. The field stop (with cross-wires, if needed) lies **in front of** the field lens.

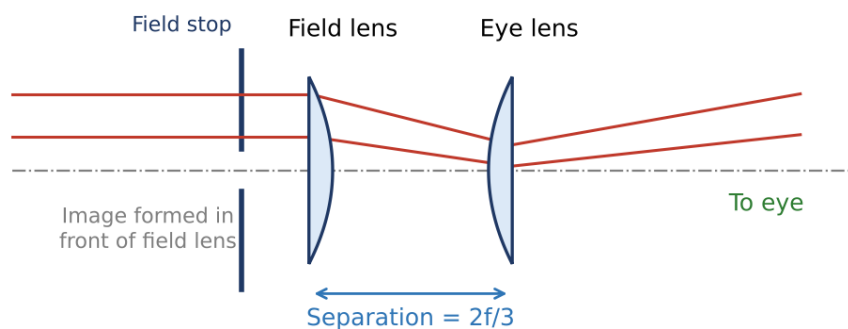


Figure 1.2: Ramsden eyepiece (a positive eyepiece with the image formed outside the lenses)

Working. For the final image at infinity, the image formed by the field lens must lie at distance f from the eye lens, i.e. $f/3$ in front of the field lens. Working back with the lens formula, the image from the objective must be at $f/4$ in front of the field lens. Thus the image lies **outside** the eyepiece, so the eyepiece is **positive** (real-image type) and a cross-wire or graticule can be placed exactly where the image forms.

Why not $d = f$? The achromatic condition $d = (f_1 + f_2)/2$ would give $d = f$, but then the intermediate image would fall on the surface of the field lens, and every scratch or speck of dust on it would be seen sharply magnified. The separation is therefore reduced to $2f/3$, which brings some residual chromatic aberration (partly compensated by using two lenses) in exchange for a clean field.

Equivalent focal length. With $f_1 = f_2 = f$ and $d = 2f/3$,

$$1/F = 2/f - (2f/3)/f^2 = 4/(3f) \Rightarrow F = 3f/4$$

Uses. Because measurement scales can be placed at the image, the Ramsden eyepiece is used in **measuring instruments**: spectrometers, travelling microscopes, comparators and optical benches.

1.5 Comparison of Huygens and Ramsden eyepieces

Point	Huygens eyepiece	Ramsden eyepiece
Type	Negative (image between the lenses)	Positive (image in front of field lens)
Focal lengths	Field lens $3f$, eye lens f	Both lenses f
Separation	$2f$	$2f/3$ (theoretically f)
Orientation of lenses	Convex faces towards the light	Convex faces towards each other
Equivalent focal length	$3f/2$	$3f/4$
Cross-wires or scale	Cannot be used	Can be used at the image
Chromatic aberration	Removed ($d = (f_1 + f_2)/2$)	Small residual error
Spherical aberration	Very small	Small
Dust on field lens	Not visible	Not visible (image in front of the lens)
Field of view	Wide	Somewhat smaller
Main use	Observation (ordinary microscopes, telescopes)	Measurement (spectrometer, travelling microscope)

1.6 Compound microscope

Construction. A compound microscope has two converging lens systems on a common axis: an **objective** of very short focal length f_o and a **eyepiece** of somewhat longer focal length f_e . The distance between the second focal point of the objective and the first focal point of the eyepiece is the **tube length** L (usually about 16 cm).

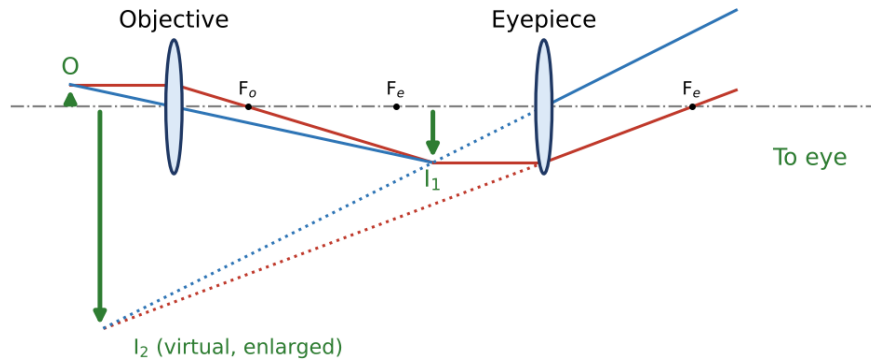


Figure 1.3: Ray diagram of a compound microscope with the final image at the least distance of distinct vision

Working. The small object O is placed just outside the first focal point of the objective. The objective forms a real, inverted and magnified image I_1 . This image lies within the focal length of the eyepiece, which therefore acts as a simple magnifier and produces a **virtual, further magnified, inverted** final image I_2 at a distance D (25 cm) from the eye.

Magnifying power

The total magnification is the product of the linear magnification of the objective and the angular magnification of the eyepiece:

$$M = m_o \times m_e$$

For the objective, with object distance u_o and image distance v_o : $m_o = v_o/u_o$. For the eyepiece used as a simple magnifier with the image at D : $m_e = 1 + D/f_e$. Hence for the final image at the least distance of distinct vision

$$M = (v_o/u_o)(1 + D/f_e)$$

When the final image is at **infinity** (normal adjustment, most relaxed eye), $m_e = D/f_e$ and, since $u_o \approx f_o$ and $v_o \approx L$,

$$M \approx (L/f_o)(D/f_e)$$

This shows that a high magnification needs a short f_o , a short f_e and a long tube.

Resolving power

The smallest separation of two points that can still be seen as separate is limited by diffraction:

$$d_{\min} = 0.61\lambda / (n \sin \theta) = 0.61\lambda / NA$$

where $n \sin \theta$ is the **numerical aperture** (NA) of the objective. Oil-immersion objectives increase n and so improve resolution.

Example 1.1: Microscope magnification

An objective of focal length 0.5 cm forms an image 16 cm from it of an object placed at 0.516 cm. If the eyepiece has focal length 5 cm, find the magnification for the final image at 25 cm.

$m_o = v_o/u_o = 16/0.516 \approx 31$. $m_e = 1 + D/f_e = 1 + 25/5 = 6$. Therefore $M = 31 \times 6 \approx 186$.

1.7 Telescope (refracting, astronomical type)

Construction. An astronomical telescope has an **objective** of large focal length f_o and large aperture and an **eyepiece** of short focal length f_e , with a common axis. Light from a distant object enters as a nearly parallel beam.

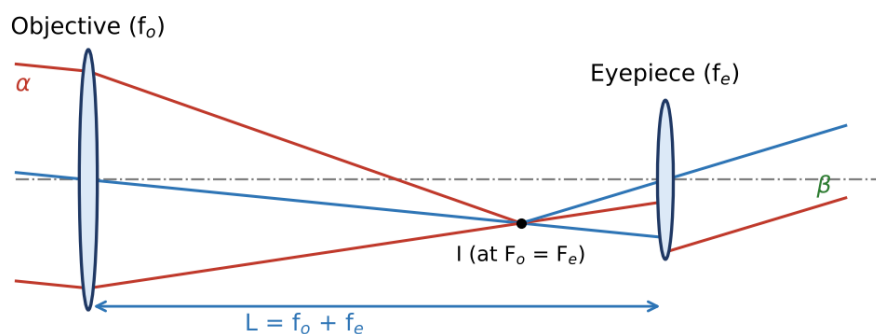


Figure 1.4: Astronomical telescope in normal adjustment (final image at infinity)

Working. The objective forms a real, inverted, diminished image of the distant object in its focal plane. The eyepiece is placed so that this image lies at (or just inside) its own focal point, and it magnifies the image. The final image is virtual and inverted.

Magnifying power

The magnifying power is the ratio of the angle β subtended at the eye by the final image to the angle α subtended by the object directly:

$$M = \beta/\alpha$$

In **normal adjustment** (final image at infinity), the image from the objective lies exactly at the focus of the eyepiece. Because $\alpha \approx h/f_o$ and $\beta \approx h/f_e$ (h = height of the first image),

$$M = f_o / f_e, \quad \text{tube length } L = f_o + f_e$$

If the final image is formed at the least distance of distinct vision D , then

$$M = (f_o/f_e)(1 + f_e/D), \quad L = f_o + u_e$$

where u_e is the distance of the intermediate image from the eyepiece.

Defects of the refracting telescope. (i) Chromatic and spherical aberration in the objective; (ii) a large lens is heavy and can only be supported at its edge, so it sags; (iii) glass must be perfect throughout; (iv) the image is inverted (acceptable in astronomy).

Example 1.2: Telescope

The focal lengths of the objective and eyepiece of a telescope are 100 cm and 5 cm. Find the magnification and tube length in normal adjustment.

$M = f_o/f_e = 100/5 = 20$. $L = f_o + f_e = 100 + 5 = 105$ cm.

1.8 Reflecting telescope

A **reflecting telescope** uses a large concave mirror, instead of a lens, as the objective. The mirror collects light from a distant object and brings it to a focus, where a small eyepiece magnifies the image.

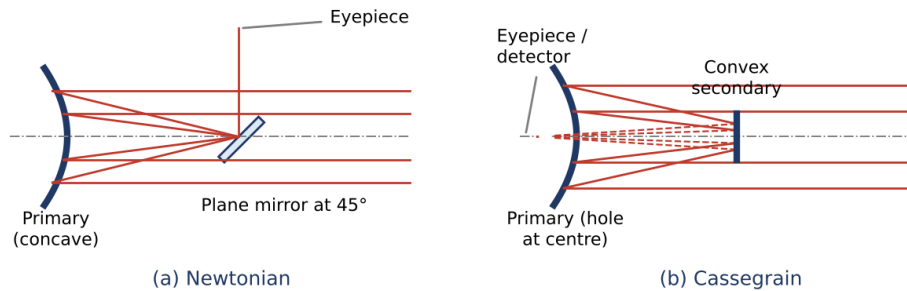


Figure 1.5: (a) Newtonian and (b) Cassegrain reflecting telescopes

Newtonian form. A paraboloidal concave mirror reflects the incoming parallel rays toward its focus. Before they reach the focus, a small **plane mirror inclined at 45°** diverts them sideways to an eyepiece fixed in the side of the tube.

Cassegrain form. The primary mirror has a hole at its centre. A small **convex secondary mirror** placed in front of the focus reflects the light back through the hole to an eyepiece or detector behind the primary mirror, giving a long effective focal length in a short tube.

Advantages over refractors

- No chromatic aberration, because reflection does not depend on wavelength.
- A paraboloidal surface removes spherical aberration.
- A mirror can be supported over its whole back surface, so very large apertures are possible, which gives greater light-gathering power and resolving power.
- Only one surface has to be ground and polished accurately, and the glass need not be optically perfect.

Disadvantages. The secondary mirror partly obstructs the beam, mirrors tarnish and need re-coating, and off-axis images show coma.

1.9 Constant deviation spectrometer

A **constant deviation spectrometer** is a spectrometer in which the angle between the collimator and the telescope is **fixed at 90°** . Instead of moving the telescope to follow each spectral line, the observer **rotates the prism** until the line of interest falls on the cross-wires; the angle of rotation is read directly in wavelength from a calibrated drum.

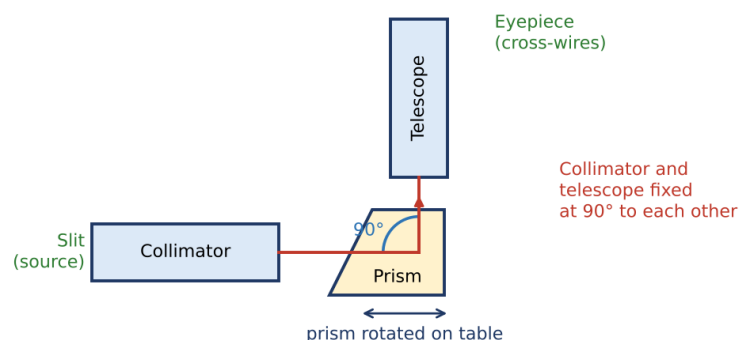


Figure 1.6: Layout of a constant deviation spectrometer

Pellin–Broca prism. The dispersing element is a single glass block of four faces, equivalent to two 30° – 60° – 90° prisms and one 45° – 45° – 90° prism cemented together. Light enters one face and is refracted; it is then **totally internally reflected** at a second face; and finally it is refracted out at a third face. The net result is that the ray of one particular wavelength, which at the first and third faces passes at minimum deviation, is turned through exactly 90° from its original direction, whatever the wavelength. Rotating the prism table brings a different wavelength into this condition.

Advantages

- Collimator and telescope remain fixed, so the instrument is compact and mechanically simple.
- Wavelength is read directly from a drum scale, so no calibration curve is needed for each measurement.
- Every wavelength is observed at minimum deviation, where the image is sharpest.

Uses. Quick determination of wavelengths, identification of elements from their line spectra, and use as a **monochromator** to select a narrow band of wavelengths.

1.10 Additional solved problems

Example 1.3: Huygens eyepiece

A Huygens eyepiece has $f = 2$ cm. Find the focal lengths of its lenses, their separation and the equivalent focal length.

$f_1 = 3f = 6$ cm, $f_2 = f = 2$ cm and $d = 2f = 4$ cm. Then $1/F = 1/6 + 1/2 - 4/(6 \times 2) = 1/6 + 3/6 - 2/6 = 2/6$, so $F = 3$ cm ($= 3f/2$).

Example 1.4: Ramsden eyepiece

Two plano-convex lenses of focal length 3 cm form a Ramsden eyepiece. Find the separation and the equivalent focal length.

$d = 2f/3 = 2$ cm. $1/F = 1/3 + 1/3 - 2/(3 \times 3) = 2/3 - 2/9 = 4/9$, so $F = 2.25$ cm ($= 3f/4$).

Example 1.5: Telescope focused on a near object

A telescope has $f_o = 100$ cm and $f_e = 5$ cm. Find M when the final image is at 25 cm.

$M = (f_o/f_e)(1 + f_e/D) = 20 \times (1 + 5/25) = 20 \times 1.2 = 24$.

1.11 Microscope and telescope compared

Feature	Compound microscope	Astronomical telescope
Purpose	View small, nearby objects	View large, distant objects
Objective	Very short focal length f_o	Long focal length f_o , large aperture
Eyepiece	Short focal length f_e	Shorter focal length f_e
Magnification (normal adjustment)	$(L/f_o)(D/f_e)$	f_o/f_e
Distance between lenses	Tube length $L + f_o + f_e$	$f_o + f_e$
Intermediate image	Real, inverted, magnified	Real, inverted, diminished
Final image	Virtual, inverted, magnified	Virtual, inverted

Key points: Unit I

- Huygens eyepiece: $f_1 = 3f$, $f_2 = f$, $d = 2f$, negative, used for observation, $F = 3f/2$.
- Ramsden eyepiece: $f_1 = f_2 = f$, $d = 2f/3$, positive, used for measurement, $F = 3f/4$.
- Achromatism of two lenses of the same material: $d = (f_1 + f_2)/2$.
- Microscope: $M = (v_o/u_o)(1 + D/f_e)$; in normal adjustment $M \approx (L/f_o)(D/f_e)$.
- Telescope in normal adjustment: $M = f_o/f_e$, $L = f_o + f_e$.
- Constant deviation spectrometer: telescope and collimator fixed at 90° ; prism rotated.

UNIT II · 08 hours

Electromagnetism

2.1 Introduction

In 1820 Oersted showed that an electric current produces a magnetic field. Eleven years later Michael Faraday (in England) and Joseph Henry (in America) discovered the reverse effect: a **changing magnetic field can produce an electric current**. This is **electromagnetic induction**, and it is the principle behind generators, transformers, induction cooking, microphones and many sensors.

The key quantity is **magnetic flux** Φ_B , the total magnetic field passing through a surface. For a uniform field B making angle θ with the normal of a plane area A :

$$\Phi_B = B A \cos \theta \quad (\text{unit: weber, } 1 \text{ Wb} = 1 \text{ T m}^2)$$

For a non-uniform field or curved surface, $\Phi_B = \int B \cdot dA$. Flux can change in three ways: B changes, the area A changes, or the angle θ changes. Any of these induces an **electromotive force (emf)** in the circuit.

2.2 Faraday's law and Lenz's law

Experimental observations

Move a bar magnet towards a coil connected to a sensitive galvanometer: the needle deflects. Hold the magnet still: no deflection. Withdraw it: the needle deflects the other way. The same happens if the coil moves instead of the magnet, or if a second coil carrying a changing current is placed nearby.

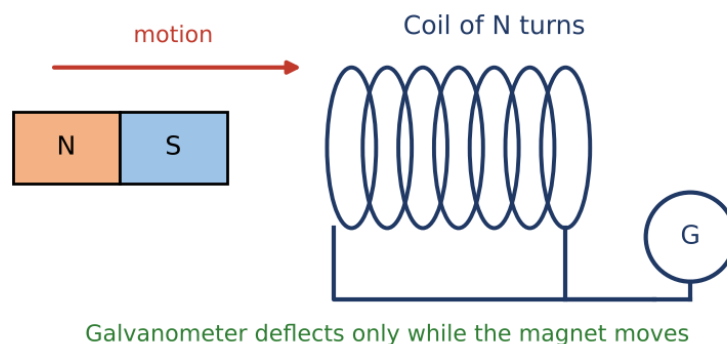


Figure 2.1: Relative motion between magnet and coil produces a deflection in the galvanometer

Faraday's laws

- **First law:** whenever the magnetic flux linked with a circuit changes, an emf is induced in it. The emf lasts only as long as the change continues.
- **Second law:** the magnitude of the induced emf equals the **rate of change of flux linkage**.

For a coil of N turns each linked with flux Φ_B ,

$$\varepsilon = -N d\Phi_B/dt$$

The product $N\Phi_B$ is the **flux linkage**. The negative sign is explained by Lenz's law.

Lenz's law

Statement: the direction of the induced current (or emf) is such that it **opposes the change** in flux that produces it.

When the north pole of a magnet approaches a coil, the induced current flows in the sense that makes the face of the coil towards the magnet a north pole, repelling the magnet. When the magnet is withdrawn, the induced current makes that face a south pole, attracting the magnet back. Either way, the induced current resists the motion.

Lenz's law and conservation of energy. If the induced current helped the change instead of opposing it, the magnet would be accelerated further, flux would change faster, and the current would grow without limit: energy would be created from nothing. The law of conservation of energy therefore requires the opposition. The work you do in pushing the magnet against this opposition is what appears as electrical energy (and heat) in the circuit.

Motional emf

A conductor of length l moving with velocity v perpendicular to a uniform field B sweeps area $l v dt$ in time dt , so the flux changes at rate $B l v$. The induced emf is

$$\varepsilon = B l v$$

Example 2.1: Induced emf

A coil of 200 turns has the flux through each turn changed from 0.05 Wb to 0.01 Wb in 0.2 s. Find the average induced emf.

$$\varepsilon = N |\Delta\Phi|/\Delta t = 200 \times (0.05 - 0.01)/0.2 = 200 \times 0.2 = \mathbf{40 \text{ V}}.$$

Example 2.2: Motional emf

A 0.5 m rod moves at 4 m s⁻¹ perpendicular to a field of 0.3 T. Find the emf.

$$\varepsilon = B l v = 0.3 \times 0.5 \times 4 = \mathbf{0.6 \text{ V}}.$$

2.3 Self-induction and self-inductance

When the current in a coil changes, the magnetic flux linked with the **same coil** changes, and so an emf is induced in it. This phenomenon is **self-induction**, and the induced emf opposes the change of current (Lenz's law); that is why a coil resists sudden changes in current, much as mass resists sudden changes in velocity.

The flux linkage is proportional to the current:

$$N \Phi_B = L I$$

The constant of proportionality L is the **self-inductance** (or coefficient of self-induction) and depends only on the geometry of the coil and the material of its core. Differentiating,

$$\varepsilon = -N d\Phi_B/dt = -L dI/dt$$

Unit. The **henry (H)**: a coil has a self-inductance of 1 H if an emf of 1 V is induced in it when the current changes at the rate 1 A s⁻¹. $1 \text{ H} = 1 \text{ V s A}^{-1} = 1 \text{ Wb A}^{-1}$.

Energy stored in an inductor

To raise the current from 0 to I against the back emf, the source does work $dW = L i di$. Integrating,

$$U = \int_0^I L i di = \frac{1}{2} L I^2$$

This energy is stored in the magnetic field of the coil and returned to the circuit when the current falls.

2.4 Self-inductance of a coil and of a solenoid

Self-inductance of a long solenoid

Consider a long air-cored solenoid of length l , cross-sectional area A and N turns (turns per unit length $n = N/l$), carrying current I .

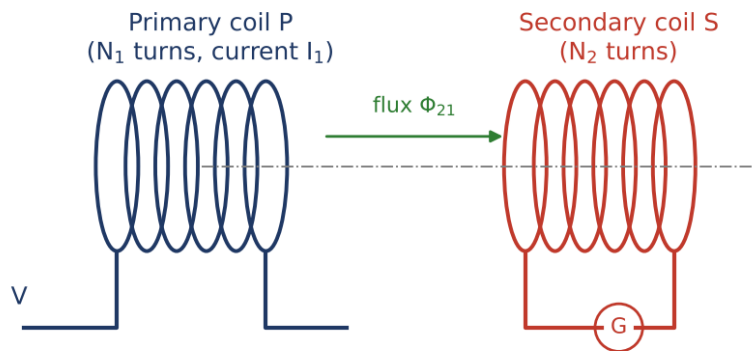


Figure 2.2: Two co-axial coils; the changing current in the primary links flux Φ_{21} with the secondary

By Ampère's law the field inside a long solenoid is uniform:

$$B = \mu_0 n I = \mu_0 N I / l$$

The flux through one turn is $\Phi_B = B A = \mu_0 N I A / l$, so the flux linkage is

$$N \Phi_B = \mu_0 N^2 A I / l$$

Comparing with $N \Phi_B = L I$,

$$L = \mu_0 N^2 A / l$$

If the solenoid is wound on a core of relative permeability μ_r , then $L = \mu_0 \mu_r N^2 A / l$.

Result: $L \propto N^2$ (doubling the turns quadruples L), $L \propto A$, $L \propto 1/l$, and an iron core multiplies L by μ_r (hundreds or thousands).

Self-inductance of a coil (general)

For any coil, $L = N \Phi_B / I$. For a circular coil it can be found if the field produced by the coil on its own cross-section is known. In practice L is measured with an inductance bridge, and **long solenoid formula** above is used for design estimates, since for a short coil the field is not uniform and the formula is only approximate.

Example 2.3: Self-inductance of a solenoid

A solenoid has 500 turns, length 0.5 m and cross-sectional area $4 \times 10^{-4} \text{ m}^2$. Find L .

$$L = \mu_0 N^2 A / l = (4\pi \times 10^{-7} \times 500^2 \times 4 \times 10^{-4}) / 0.5 = (4\pi \times 10^{-7} \times 2.5 \times 10^5 \times 4 \times 10^{-4}) / 0.5 \approx 2.5 \times 10^{-4} \text{ H} = \mathbf{0.25 \text{ mH}}.$$

Example 2.4: Back emf and stored energy

(a) A 5 H inductor has the current changing at 2 A s^{-1} . (b) A 2 H inductor carries 3 A. Find the induced emf in (a) and energy in (b).

$$(a) \mathcal{E} = L \, dI/dt = 5 \times 2 = \mathbf{10 \text{ V}}. \quad (b) U = \frac{1}{2} L I^2 = \frac{1}{2} \times 2 \times 3^2 = \mathbf{9 \text{ J}}.$$

2.5 Mutual induction and mutual inductance

When two coils are close to each other, a changing current in one (the **primary**) produces a changing flux that links the other (the **secondary**), and an emf is induced in the secondary. This is **mutual induction**.

If the primary current is I_1 and it produces flux Φ_{21} through each turn of the secondary (N_2 turns), the flux linkage in the secondary is proportional to I_1 :

$$N_2 \Phi_{21} = M I_1$$

The constant M is the **mutual inductance**. The induced emf in the secondary is

$$\varepsilon_2 = -N_2 d\Phi_{21}/dt = -M dI_1/dt$$

Properties

- Unit: henry. $M = 1 \text{ H}$ if a rate of change of primary current of 1 A s^{-1} induces 1 V in the secondary.
- Reciprocity: $M_{12} = M_{21} = M$, so it does not matter which coil is the primary.
- M depends on the number of turns, the sizes of both coils, their separation and relative orientation, and the medium (core).
- **Coupling coefficient:** $M = k \sqrt{L_1 L_2}$, where $0 \leq k \leq 1$. For perfect coupling (all flux of one coil links the other), $k = 1$.

Applications. Transformers, induction coils (spark coils), wireless chargers, metal detectors.

2.6 Mutual inductance of co-axial coils and co-axial solenoids

(a) Two co-axial solenoids

Let a long solenoid 1 have N_1 turns, length l and area A , and let a short coil 2 of N_2 turns be wound tightly on its middle, so that both share the same cross-sectional area A . With current I_1 in solenoid 1, the field inside is $B = \mu_0 N_1 I_1 / l$ and the flux linked with each turn of solenoid 2 is $\Phi_{21} = BA$. Therefore

$$N_2 \Phi_{21} = \mu_0 N_1 N_2 A I_1 / l \Rightarrow M = \mu_0 N_1 N_2 A / l$$

Check with coupling: $L_1 = \mu_0 N_1^2 A / l$ and $L_2 = \mu_0 N_2^2 A / l$, so $\sqrt{L_1 L_2} = \mu_0 N_1 N_2 A / l = M$. Hence $k = 1$, as expected when all the flux of one solenoid links the other.

(b) A pair of co-axial circular coils

Take a large coil 1 (radius a , N_1 turns) and a small coil 2 (radius $b \ll a$, N_2 turns) with the same axis, their centres a distance x apart. The field on the axis of coil 1 at distance x is

$$B = \mu_0 N_1 I_1 a^2 / [2 (a^2 + x^2)^{3/2}]$$

Since b is small, B is almost uniform over coil 2, and the flux linkage is $N_2 B \pi b^2$. Hence

$$M = \mu_0 \pi N_1 N_2 a^2 b^2 / [2 (a^2 + x^2)^{3/2}]$$

M falls rapidly as the separation x increases, and is maximum when $x = 0$ (the coils in the same plane).

Example 2.5: Mutual inductance

A long solenoid has 1000 turns, length 0.4 m and area $5 \times 10^{-4} \text{ m}^2$. A coil of 200 turns is wound tightly around it. Find M and the emf in the coil if the current in the solenoid changes at 50 A s^{-1} .

$M = \mu_0 N_1 N_2 A / l = (4\pi \times 10^{-7} \times 1000 \times 200 \times 5 \times 10^{-4}) / 0.4 \approx 3.14 \times 10^{-4} \text{ H}$. $\text{emf} = M dI/dt = 3.14 \times 10^{-4} \times 50 \approx 1.57 \times 10^{-2} \text{ V} = 15.7 \text{ mV}$.

2.7 Self-induction and mutual induction compared

Feature	Self-induction	Mutual induction
Coils involved	One coil	Two neighbouring coils
Cause	Change of current in the same coil	Change of current in the other coil
Induced emf	$\varepsilon = -L \, di/dt$	$\varepsilon_2 = -M \, di_1/dt$
Depends on	Turns, area, length, core	Both coils, their separation, orientation and core
Energy	$\frac{1}{2}LI^2$	$\frac{1}{2}L_1i_1^2 + \frac{1}{2}L_2i_2^2 + M i_1i_2$ for two coupled circuits
Unit	henry (H)	henry (H)

2.8 Applications of electromagnetic induction

- **A.C. generator (alternator):** a coil of N turns and area A rotating at angular speed ω in a field B has flux $\Phi = BA \cos \omega t$, so $\varepsilon = NBA\omega \sin \omega t$. The peak emf is $\varepsilon_{\max} = NBA\omega$.
- **Transformer:** works on mutual induction; for an ideal transformer $V_s/V_p = N_s/N_p = I_p/I_s$. A step-up transformer has $N_s > N_p$.
- **Inductors and chokes:** self-induction smooths and filters current in power supplies and radio circuits.
- **Eddy currents:** induced currents circulating in bulk metal; they heat the metal (induction cooktops, furnaces) and brake moving conductors (magnetic braking), but are reduced by laminating transformer cores.
- **Other uses:** microphones and guitar pick-ups, wireless phone chargers, metal detectors, induction coils.

Example 2.6: Generator and transformer

(a) A coil of 100 turns and area $0.02 \, \text{m}^2$ rotates at 50 Hz in a field of 0.5 T. Find the peak emf. (b) A transformer has 1000 primary turns and 100 secondary turns and 220 V is applied to the primary. Find the secondary voltage.

(a) $\omega = 2\pi \times 50 = 314 \, \text{rad s}^{-1}$, $\varepsilon_{\max} = NBA\omega = 100 \times 0.5 \times 0.02 \times 314 \approx \mathbf{314 \, V}$. (b) $V_s = V_p \times N_s/N_p = 220 \times 100/1000 = \mathbf{22 \, V}$.

Key points: Unit II

- Faraday: $\varepsilon = -N \, d\Phi/dt$. Lenz: induced current opposes the change in flux (conservation of energy).
- Self-inductance: $\varepsilon = -L \, di/dt$; energy $U = \frac{1}{2}LI^2$; solenoid $L = \mu_0 N^2 A/l$.
- Mutual inductance: $\varepsilon_2 = -M \, di_1/dt$; $M_{12} = M_{21}$; coaxial solenoids $M = \mu_0 N_1 N_2 A/l$.
- Coupling: $M = k\sqrt{L_1 L_2}$, $0 \leq k \leq 1$. Unit of L and M : henry.

UNIT III · 08 hours

Electrical Circuit Analysis

3.1 Concepts of R, L and C elements

Every electrical network is built from **sources** (batteries, generators) and **passive elements**: the resistor, the inductor and the capacitor. A passive element does not generate energy; it either dissipates it (resistor) or stores it (inductor in a magnetic field, capacitor in an electric field). We use the following vocabulary when analysing circuits:

- **Node**: a point where two or more elements join. **Branch**: a path between two nodes containing one or more elements.
- **Loop**: any closed path in the network. **Mesh**: a loop that contains no other loop inside it.
- **Linear element**: its voltage–current relation is a straight line (superposition holds). **Bilateral element**: it behaves the same for either direction of current.

The three elements

Element	Symbol and unit	Voltage–current relation	Energy	Behaviour in steady DC
Resistor	R, ohm (Ω)	$v = iR$	Dissipated as heat, $p = i^2R$	Obeys Ohm's law
Inductor	L, henry (H)	$v = L \, di/dt$	Stored, $\frac{1}{2}Li^2$ (magnetic)	Acts as a short circuit ($v = 0$)
Capacitor	C, farad (F)	$i = C \, dv/dt$	Stored, $\frac{1}{2}Cv^2$ (electric)	Acts as an open circuit ($i = 0$)

Resistance arises from collisions of charge carriers with the lattice; for a uniform wire $R = \rho l/A$, where ρ is the resistivity. **Inductance** (Unit II) opposes a **change of current**, and **capacitance** opposes a **change of voltage**. In an a.c. circuit these appear as reactances $X_L = \omega L$ and $X_C = 1/(\omega C)$.

Combinations of L and C

Combination	Inductors	Capacitors
Series	$L = L_1 + L_2 + \dots$	$1/C = 1/C_1 + 1/C_2 + \dots$
Parallel	$1/L = 1/L_1 + 1/L_2 + \dots$	$C = C_1 + C_2 + \dots$

3.2 Ohm's law

Statement: at constant temperature, the current through a conductor is directly proportional to the potential difference across its ends.

$$V = I R \quad (R = \text{constant of proportionality, in ohm})$$

A graph of V against I is a straight line through the origin for an **ohmic** conductor such as a metal wire. Devices such as diodes, thermistors and filament lamps are **non-ohmic**: their V–I graph is not a straight

line, so R changes with V or temperature. The microscopic form of Ohm's law is $J = \sigma E$, where J is current density, E is the electric field and σ is the conductivity.

Power and energy. The power dissipated is $P = VI = I^2R = V^2/R$. In time t the energy converted to heat is $W = I^2Rt$ (Joule's law).

3.3 Kirchhoff's laws

Ohm's law is enough for a single loop, but a network with several sources and branches needs two further laws, which express conservation of charge and conservation of energy.

Kirchhoff's laws

- **Current law (KCL, junction rule):** the algebraic sum of the currents at a node is zero: $\sum I = 0$. Current entering a junction equals current leaving it (conservation of charge).
- **Voltage law (KVL, loop rule):** the algebraic sum of the emfs and potential drops around any closed loop is zero: $\sum E - \sum IR = 0$ (conservation of energy).

Sign conventions. Assign a direction to every branch current (a wrong guess simply gives a negative answer). Going around a loop, a potential drop $-IR$ is counted when we move along the current in a resistor; an emf is positive when we pass from the negative to the positive terminal of the cell.

Procedure

1. Mark branch currents with assumed directions and label every node.
2. Write KCL at the independent nodes (one fewer than the number of nodes).
3. Write KVL around enough independent loops to have as many equations as unknown currents.
4. Solve the simultaneous equations; a negative result means the current flows opposite to the assumed direction.

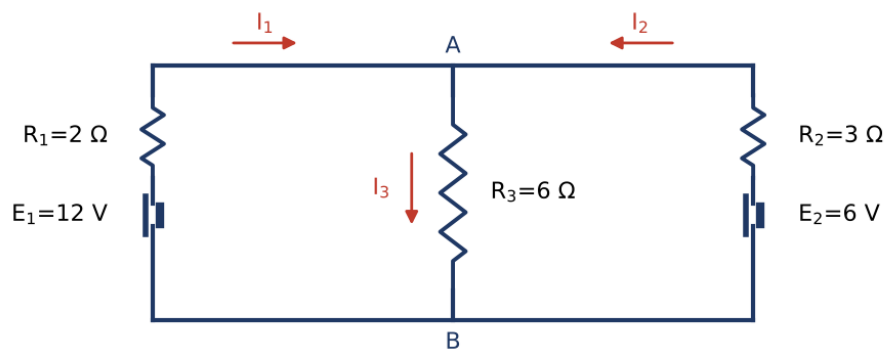


Figure 3.1: A two-battery network used in Examples 3.1, 3.3 and 3.5

Example 3.1: Kirchhoff's laws

In Figure 3.1, $E_1 = 12\text{ V}$, $R_1 = 2\ \Omega$, $E_2 = 6\text{ V}$, $R_2 = 3\ \Omega$ and $R_3 = 6\ \Omega$. Find all branch currents.

KCL at A: $I_3 = I_1 + I_2$. KVL, left loop: $12 - 2I_1 - 6I_3 = 0$, which becomes $8I_1 + 6I_2 = 12$. KVL, right loop: $6 - 3I_2 - 6I_3 = 0$, which becomes $6I_1 + 9I_2 = 6$.

Solving: $I_2 = -2/3\text{ A} \approx -0.67\text{ A}$, $I_1 = 2\text{ A}$, $I_3 = 4/3\text{ A} \approx 1.33\text{ A}$. The negative sign shows that I_2 actually flows opposite to the assumed direction, i.e. the 6 V cell is being charged by the 12 V cell. Check in the right loop: $6 - 3(-0.667) - 6(1.333) = 6 + 2 - 8 = 0\ \checkmark$.

3.4 Resistors in series and parallel

Series

Resistors are in series when the **same current** flows through each. The total voltage is the sum of the individual voltages, so

$$R_s = R_1 + R_2 + R_3 + \dots$$

The voltage divides in proportion to resistance: $V_1 = V \cdot R_1/(R_1 + R_2)$.

Parallel

Resistors are in parallel when the **same voltage** appears across each. The total current is the sum of branch currents, so

$$1/R_p = 1/R_1 + 1/R_2 + 1/R_3 + \dots$$

For two resistors $R_p = R_1 R_2 / (R_1 + R_2)$. The current divides inversely to resistance: $I_1 = I \cdot R_2 / (R_1 + R_2)$. The equivalent resistance is always **smaller than the smallest** branch resistance.

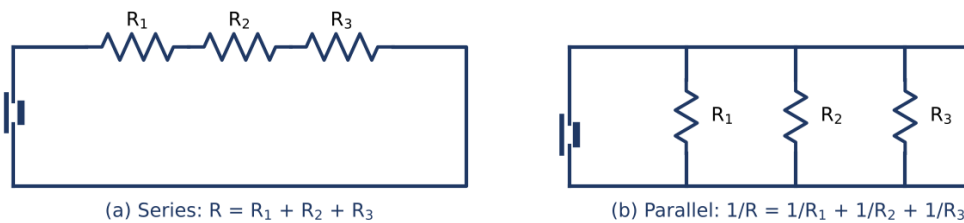


Figure 3.2: (a) series and (b) parallel combinations of resistors

Example 3.2: Equivalent resistance

Three resistors of $2\ \Omega$, $3\ \Omega$ and $6\ \Omega$ are connected (a) in series and (b) in parallel. Find the equivalent resistance in each case.

(a) $R_s = 2 + 3 + 6 = 11\ \Omega$. (b) $1/R_p = 1/2 + 1/3 + 1/6 = (3 + 2 + 1)/6 = 1$, so $R_p = 1\ \Omega$.

3.5 Superposition theorem

Statement: in a **linear, bilateral** network containing more than one source, the current through (or voltage across) any element equals the **algebraic sum** of the currents (or voltages) produced by each source acting alone, with all the other independent sources replaced by their internal resistances (voltage sources short-circuited, current sources open-circuited).

Procedure

1. Keep one source; replace each other voltage source by a short circuit and each other current source by an open circuit.
2. Find the required current or voltage due to that single source.
3. Repeat for every source in turn.
4. Add the results algebraically, paying attention to directions.

Limitation: it works only for linear networks and cannot be used directly for power, since power depends on the square of the current.

Example 3.3: Superposition

Find the current in R_3 of Figure 3.1 using superposition.

E₁ alone (E₂ shorted): $R_2 \parallel R_3 = 3 \times 6/9 = 2 \Omega$. Total resistance = $2 + 2 = 4 \Omega$, so the source current is $12/4 = 3 \text{ A}$. The share through $R_3 = 3 \times 3/(3 + 6) = 1 \text{ A}$.

E₂ alone (E₁ shorted): $R_1 \parallel R_3 = 2 \times 6/8 = 1.5 \Omega$. Total = $3 + 1.5 = 4.5 \Omega$, current = $6/4.5 = 1.333 \text{ A}$. The share through $R_3 = 1.333 \times 2/(2 + 6) = 0.333 \text{ A}$, in the same direction as before.

Total $I_3 = 1 + 0.333 = \mathbf{1.33 \text{ A}}$, matching Example 3.1.

3.6 Thevenin's theorem

Statement: any linear two-terminal network can be replaced, as seen from its terminals, by a single **voltage source V_{Th}** in series with a **resistance R_{Th}** . V_{Th} is the open-circuit voltage across the terminals, and R_{Th} is the resistance seen looking back into the terminals with all independent sources set to zero.

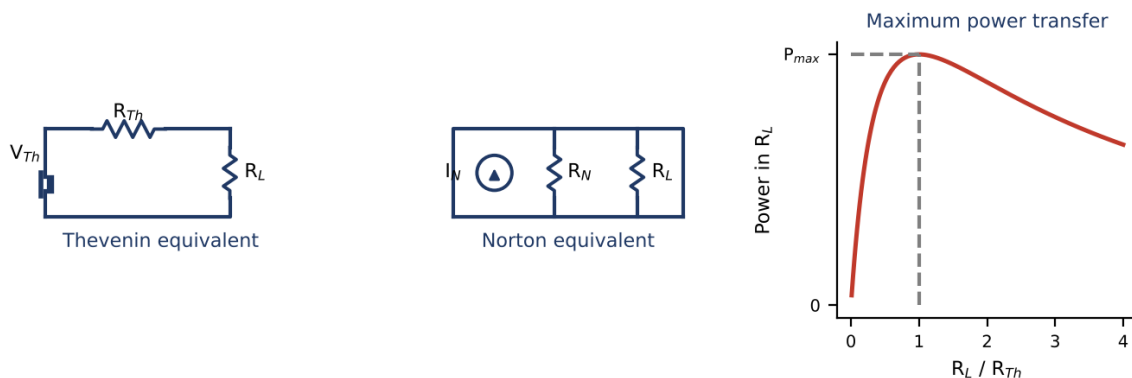


Figure 3.3: Thevenin and Norton equivalent circuits, and variation of load power with R_L

Procedure

1. Remove the load resistor R_L and mark the two terminals A and B.
2. Calculate the open-circuit voltage between A and B: this is V_{Th} .
3. Replace all independent sources by their internal resistances (short voltage sources, open current sources) and compute the resistance between A and B: this is R_{Th} .
4. Redraw the circuit as V_{Th} in series with R_{Th} and the load R_L , and find the load current $I_L = V_{Th}/(R_{Th} + R_L)$.

Example 3.4: Thevenin equivalent

A 12 V source with series resistance 4Ω feeds terminals A–B across which a 6Ω resistor is connected as a shunt. Find the load current for a load of 3.6Ω connected at A–B.

With the load removed, $V_{Th} = 12 \times 6/(4 + 6) = \mathbf{7.2 \text{ V}}$. With the source shorted, $R_{Th} = 4 \parallel 6 = 24/10 = \mathbf{2.4 \Omega}$.

$I_L = V_{Th}/(R_{Th} + R_L) = 7.2/(2.4 + 3.6) = \mathbf{1.2 \text{ A}}$.

3.7 Norton's theorem

Statement: any linear two-terminal network can be replaced by a **constant current source I_N** in parallel with a **resistance R_N** . I_N is the **short-circuit current** between the terminals, and R_N equals R_{Th} , the resistance seen from the terminals with the sources set to zero.

Procedure

1. Short the terminals A and B (remove R_L) and find the short-circuit current: this is I_N .
2. Find R_N exactly as R_{Th} was found.

3. Connect the load across the Norton equivalent; the load current is $I_L = I_N \cdot R_N / (R_N + R_L)$ (current division).

Relation between Thevenin and Norton forms

Quantity	Thevenin to Norton	Norton to Thevenin
Source	$I_N = V_{Th}/R_{Th}$	$V_{Th} = I_N R_N$
Resistance	$R_N = R_{Th}$	$R_{Th} = R_N$

Example 3.5: Norton equivalent

For the circuit of Example 3.4, find I_N , R_N and the load current.

$I_N = V_{Th}/R_{Th} = 7.2/2.4 = 3 \text{ A}$, $R_N = 2.4 \Omega$. Load current $I_L = 3 \times 2.4/(2.4 + 3.6) = 1.2 \text{ A}$, the same as the Thevenin result.

3.8 Maximum power transfer theorem

Statement: a voltage source with internal (Thevenin) resistance R_{Th} delivers the **maximum power to a load when the load resistance equals the source resistance**, $R_L = R_{Th}$.

Proof. Current in the load $I = V_{Th}/(R_{Th} + R_L)$, so the power in the load is

$$P = I^2 R_L = V_{Th}^2 R_L / (R_{Th} + R_L)^2$$

For maximum power, $dP/dR_L = 0$:

$$dP/dR_L = V_{Th}^2 [(R_{Th} + R_L)^2 - 2R_L(R_{Th} + R_L)] / (R_{Th} + R_L)^4 = 0$$

This requires $R_{Th} + R_L = 2R_L$, i.e. $R_L = R_{Th}$. The second derivative is negative here, so it is a maximum. Substituting back,

$$P_{max} = V_{Th}^2 / (4 R_{Th})$$

Efficiency. At the matching condition the same power is dissipated in the source resistance as in the load, so the efficiency is only **50 %**. Matching is therefore used when the signal power is small and the aim is the largest signal (audio amplifiers to loudspeakers, antennas), but not in power transmission, where high efficiency is essential.

Example 3.6: Maximum power

For the Thevenin equivalent of Example 3.4 ($V_{Th} = 7.2 \text{ V}$, $R_{Th} = 2.4 \Omega$), find the load for maximum power and that power.

$R_L = R_{Th} = 2.4 \Omega$. $P_{max} = 7.2^2/(4 \times 2.4) = 51.84/9.6 = 5.4 \text{ W}$.

3.9 Choosing the right method

Method	Best used when	Result
Kirchhoff's laws	The network has several loops and every branch current is needed	All branch currents
Superposition	The network is linear and has several independent sources	Sum of the effects of each source alone
Thevenin	Only one element (the load) changes and a voltage source is natural	V_{Th} in series with R_{Th}

Method	Best used when	Result
Norton	A current source is more natural (parallel-type circuits)	I_N in parallel with R_N
Maximum power transfer	The load can be chosen to match the source	$R_L = R_{Th}$, $P_{max} = V_{Th}^2/(4R_{Th})$

Source transformation

A voltage source V in series with a resistance R is equivalent, at its terminals, to a current source $I = V/R$ in parallel with the same R . This allows series and parallel sources to be converted back and forth to simplify a network, and it is the idea that links Thevenin and Norton equivalents.

3.10 Current and voltage division

Example 3.7: Current divider

A current of 6 A enters two parallel resistors of 4 Ω and 12 Ω . Find the current in each.

$I_4 = 6 \times 12/(4 + 12) = \mathbf{4.5\ A}$; $I_{12} = 6 \times 4/(4 + 12) = \mathbf{1.5\ A}$. The smaller resistor carries the larger current, and the two currents add up to 6 A.

Example 3.8: Voltage divider

A 24 V supply is connected across 6 Ω and 18 Ω in series. Find the voltage across the 18 Ω resistor.

$V_{18} = 24 \times 18/(6 + 18) = \mathbf{18\ V}$; the remaining 6 V appears across the 6 Ω resistor.

Key points: Unit III

- $R: v = iR$; $L: v = L di/dt$; $C: i = C dv/dt$. In steady DC, L acts as a short, C as an open circuit.
- KCL: $\sum I = 0$ at a node. KVL: $\sum E = \sum IR$ around a loop.
- Series R : $R = \sum R_n$. Parallel R : $1/R = \sum 1/R_n$.
- Superposition: add the effects of each source acting alone (linear networks only).
- Thevenin: V_{Th} (open circuit) in series with R_{Th} . Norton: I_N (short circuit) in parallel with $R_N = R_{Th}$.
- Maximum power: $R_L = R_{Th}$, $P_{max} = V_{Th}^2/(4R_{Th})$, efficiency 50 %.

UNIT IV · 07 hours

Kinetic Theory of Matter

4.1 Introduction, three states of matter, ideal gas and kinetic model

Matter is made of atoms and molecules that are in constant motion and exert forces on one another. The **kinetic theory** explains the large-scale (macroscopic) properties of matter such as pressure and temperature in terms of the average behaviour of these tiny particles. It links the microscopic world of molecules to the quantities we measure in the laboratory.

Three states of matter

Property	Solid	Liquid	Gas
Shape and volume	Fixed shape, fixed volume	Takes shape of container, fixed volume	Fills container completely, no fixed shape or volume
Intermolecular forces	Very strong	Moderate	Very weak (negligible)
Molecular arrangement	Regular, closely packed	Close but disordered	Far apart, random
Molecular motion	Vibration about fixed positions	Vibration, rotation and sliding	Free, rapid, random translation
Compressibility	Almost nil	Very small	Large

Heating a solid increases the amplitude of vibration until the lattice melts into a liquid; further heating lets molecules escape into the vapour. The same molecules can therefore exist in all three states, the difference being the balance between kinetic energy (which drives molecules apart) and intermolecular attraction (which holds them together).

Ideal gas

An **ideal (perfect) gas** is one that obeys the gas laws exactly at all temperatures and pressures. Real gases approach this behaviour at **low pressure and high temperature**, when molecules are far apart and their forces and sizes become unimportant. Hydrogen, helium, nitrogen and oxygen at room conditions are very nearly ideal.

Kinetic model: assumptions

Postulates of the kinetic theory of gases

- A gas consists of a very large number of identical molecules, each of mass m , which are very small compared with the distances between them.
- The molecules are in continuous, random motion in all directions with all possible speeds.
- Collisions between molecules, and with the walls, are **perfectly elastic**, so the kinetic energy is conserved.
- There are no attractive or repulsive forces between molecules except during collisions, so between collisions each moves in a straight line at constant speed (Newton's first law).
- The time of a collision is negligible compared with the time between collisions.
- The molecules are distributed uniformly through the container, and gravity may be neglected.
- The average kinetic energy of a molecule is proportional to the absolute temperature of the gas.

4.2 Pressure exerted by a gas and r.m.s. speed of molecules

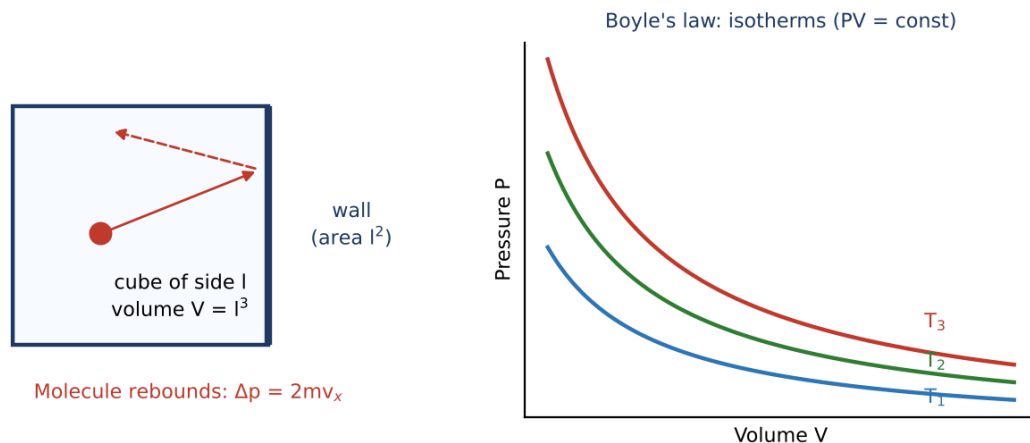


Figure 4.1: A molecule rebounding from a wall of the cube, and isotherms illustrating Boyle's law

Derivation of the pressure

Consider a cubical vessel of side l containing N molecules, each of mass m . Take one molecule with velocity components (c_x, c_y, c_z) , so that $c^2 = c_x^2 + c_y^2 + c_z^2$.

1. **Change of momentum.** The molecule strikes the wall perpendicular to the x -axis with momentum $+mc_x$ and rebounds elastically with $-mc_x$. The change of momentum is $2mc_x$.
2. **Collision frequency.** It travels across the cube and back, a distance $2l$, in time $2l/c_x$, so it hits this wall $c_x/(2l)$ times per second.
3. **Force from one molecule.** Rate of change of momentum $= 2mc_x \times c_x/(2l) = m c_x^2/l$.
4. **Force from all molecules.** $F = (m/l)(c_{1x}^2 + c_{2x}^2 + \dots + c_{Nx}^2) = (mN/l) \langle c_x^2 \rangle$, where $\langle c_x^2 \rangle$ is the mean value.
5. **Isotropy.** The motion is random, so $\langle c_x^2 \rangle = \langle c_y^2 \rangle = \langle c_z^2 \rangle = \langle c^2 \rangle/3$.
6. **Pressure.** $P = F/l^2 = (mN/3l^3)\langle c^2 \rangle$, and since $V = l^3$,

$$P = (1/3)(mN/V)\langle c^2 \rangle = (1/3) \rho \langle c^2 \rangle$$

where $\rho = mN/V$ is the density of the gas. Although we used a cube, the result is true for a vessel of any shape.

Root mean square (r.m.s.) speed

The r.m.s. speed is defined as $c_{rms} = \sqrt{\langle c^2 \rangle}$, the square root of the mean of the squares of the molecular speeds. From the pressure equation,

$$c_{rms} = \sqrt{(3P/\rho)} = \sqrt{(3kT/m)} = \sqrt{(3RT/M)}$$

where k is Boltzmann's constant, R the gas constant and M the molar mass. At the same temperature, lighter molecules move faster: hydrogen at 300 K has $c_{rms} \approx 1930 \text{ m s}^{-1}$, oxygen only $\approx 484 \text{ m s}^{-1}$.

Kinetic energy and temperature

The average translational kinetic energy of a molecule is

$$\frac{1}{2} m \langle c^2 \rangle = (3/2) k T$$

so the **absolute temperature is a measure of the average kinetic energy** of the molecules. For one mole, the total translational energy is $(3/2)RT$. At absolute zero the translational motion would (classically) cease.

Example 4.1: r.m.s. speed and kinetic energy

Find the r.m.s. speed of oxygen molecules ($M = 32 \times 10^{-3} \text{ kg mol}^{-1}$) at 300 K and the mean kinetic energy per molecule.

$c_{rms} = \sqrt{(3RT/M)} = \sqrt{(3 \times 8.314 \times 300/0.032)} = \sqrt{(2.34 \times 10^5)} \approx 484 \text{ m s}^{-1}$. Mean kinetic energy = $(3/2)kT = 1.5 \times 1.38 \times 10^{-23} \times 300 \approx 6.2 \times 10^{-21} \text{ J}$.

Example 4.2: Comparing two gases

Find the ratio of the r.m.s. speeds of hydrogen and oxygen at the same temperature.

$c_H/c_O = \sqrt{(M_O/M_H)} = \sqrt{(32/2)} = \sqrt{16} = 4$: hydrogen molecules are four times faster.

4.3 Deduction of Boyle's law

Statement. At constant temperature, the pressure of a fixed mass of gas is inversely proportional to its volume: $P \propto 1/V$, or $PV = \text{constant}$.

Deduction. From the kinetic equation, with N molecules of mass m ,

$$PV = (1/3) m N \langle c^2 \rangle = (2/3) N (\frac{1}{2} m \langle c^2 \rangle)$$

By postulate, the mean kinetic energy $\frac{1}{2}m\langle c^2 \rangle$ depends only on the temperature. For a fixed mass of gas, N is constant. Therefore, if T is constant, the right-hand side is constant:

$$PV = \text{constant} \quad (T \text{ and mass constant})$$

This is **Boyle's law**. On a graph of P against V at fixed temperature the curve is a rectangular hyperbola (an **isotherm**), and a graph of P against $1/V$ is a straight line through the origin. Higher temperature shifts the whole curve outwards (Figure 4.1).

4.4 Derivation of the gas equation

From the kinetic equation $PV = (2/3)N(\frac{1}{2}m\langle c^2 \rangle)$ and the relation $\frac{1}{2}m\langle c^2 \rangle = (3/2)kT$,

$$PV = (2/3) N (3/2) k T = N k T$$

If the gas contains n moles, $N = nN_A$, so $PV = n N_A k T$. The product $N_A k$ is the **universal gas constant** R :

$$PV = n R T \quad R = N_A k = 8.314 \text{ J mol}^{-1} \text{ K}^{-1}$$

For one mole, $PV = RT$. This is the **ideal gas equation**. It combines Boyle's, Charles's and Avogadro's laws in one expression, and it explains why the Boltzmann constant $k = R/N_A = 1.38 \times 10^{-23} \text{ J K}^{-1}$ is the gas constant per molecule.

Example 4.3: Gas equation

How many molecules are in 1 m^3 of gas at pressure $1.013 \times 10^5 \text{ Pa}$ and 300 K ?

$$N = PV/(kT) = (1.013 \times 10^5 \times 1)/(1.38 \times 10^{-23} \times 300) \approx 2.4 \times 10^{25} \text{ molecules.}$$

4.5 Derivation of the gas laws from kinetic theory

All the experimental gas laws follow from $P = (1/3)(mN/V)\langle c^2 \rangle$ and $\frac{1}{2}m\langle c^2 \rangle = (3/2)kT$.

(a) Charles's law

Statement. At constant pressure, the volume of a fixed mass of gas is directly proportional to its absolute temperature: $V \propto T$.

Deduction. From $PV = NkT$, $V = (Nk/P)T$. For fixed N and constant P , $V \propto T$. In terms of the Celsius scale, volume increases by $1/273$ of its value at 0°C for each 1°C rise.

(b) Gay-Lussac's (pressure) law

Statement. At constant volume, the pressure of a fixed mass of gas is directly proportional to its absolute temperature: $P \propto T$.

Deduction. From $PV = NkT$, $P = (Nk/V)T$; with N and V constant, $P \propto T$.

(c) Avogadro's law

Statement. Equal volumes of all gases at the same temperature and pressure contain the same number of molecules.

Deduction. For two gases with the same P and V , $PV = (2/3)N_1(\frac{1}{2}m_1\langle c_1^2 \rangle) = (2/3)N_2(\frac{1}{2}m_2\langle c_2^2 \rangle)$. At the same temperature the mean kinetic energies per molecule are equal, $\frac{1}{2}m_1\langle c_1^2 \rangle = \frac{1}{2}m_2\langle c_2^2 \rangle$, so $N_1 = N_2$. At STP one mole (6.022×10^{23} molecules) occupies 22.4 litres.

(d) Dalton's law of partial pressures

Statement. The total pressure of a mixture of non-reacting gases is the sum of the partial pressures that each gas would exert if it alone occupied the container at the same temperature.

Deduction. For a mixture of gases with $N_1, N_2, \dots$ molecules of masses $m_1, m_2, \dots$ in volume V :

$$P = (1/3V)[N_1m_1\langle c_1^2 \rangle + N_2m_2\langle c_2^2 \rangle + \dots] = P_1 + P_2 + \dots$$

because molecules do not interact and each gas contributes to the pressure independently.

(e) Graham's law of diffusion

Statement. At constant temperature and pressure, the rate of diffusion of a gas is inversely proportional to the square root of its density (or molar mass).

Deduction. From $c_{\text{rms}} = \sqrt{3P/\rho}$, the speed of the molecules, and hence the rate of diffusion, is proportional to $1/\sqrt{\rho}$. For two gases at the same P and T : $r_1/r_2 = \sqrt{\rho_2/\rho_1} = \sqrt{M_2/M_1}$.

Law	Condition kept constant	Result	Kinetic origin
Boyle	$T, \text{ mass}$	$PV = \text{constant}$	Mean KE fixed, so PV fixed

Law	Condition kept constant	Result	Kinetic origin
Charles	P, mass	$V \propto T$	$V = NkT/P$
Gay-Lussac	V, mass	$P \propto T$	$P = NkT/V$
Avogadro	P, V, T	$N_1 = N_2$	Equal mean KE at equal T
Dalton	T, V	$P = \sum P_i$	Molecules act independently
Graham	T, P	$r \propto 1/\sqrt{M}$	$c_{rms} \propto 1/\sqrt{M}$

4.6 Real gases compared with ideal gases

Feature	Ideal gas	Real gas
Intermolecular forces	None except in collisions	Attractive at moderate distances, repulsive when very close
Size of molecules	Negligible	Finite
Gas equation	$PV = nRT$ at all P and T	Approximately $PV = nRT$ at low P and high T; van der Waals: $(P + a/V^2)(V - b) = RT$ per mole
Liquefaction	Not possible	Possible below the critical temperature

4.7 Additional solved problems

Example 4.4: Boyle's law

A gas at 2 atm occupies 3 litres. It is compressed isothermally to 1.5 litres. Find the new pressure.

$P_1V_1 = P_2V_2$, so $P_2 = 2 \times 3/1.5 = \mathbf{4 \text{ atm}}$.

Example 4.5: Charles's law

A gas occupies 500 cm³ at 27 °C. What is its volume at 127 °C at constant pressure?

$T_1 = 300 \text{ K}$, $T_2 = 400 \text{ K}$. $V_2 = V_1T_2/T_1 = 500 \times 400/300 = \mathbf{667 \text{ cm}^3}$.

Example 4.6: Graham's law

Compare the rates of diffusion of hydrogen ($M = 2$) and carbon dioxide ($M = 44$) under the same conditions.

$r_H/r_{CO_2} = \sqrt{44/2} = \sqrt{22} \approx \mathbf{4.7}$: hydrogen diffuses about 4.7 times faster.

Example 4.7: Dalton's law

A vessel contains 0.2 mol of oxygen and 0.3 mol of nitrogen at a total pressure of 5 atm. Find the partial pressures.

Partial pressure is proportional to the mole fraction: $P(O_2) = 5 \times 0.2/0.5 = \mathbf{2 \text{ atm}}$, $P(N_2) = 5 \times 0.3/0.5 = \mathbf{3 \text{ atm}}$; the sum is 5 atm.

Key points: Unit IV

- $P = (1/3)(mN/V)\langle c^2 \rangle = (1/3)\rho\langle c^2 \rangle$; $c_{rms} = \sqrt{3kT/m} = \sqrt{3RT/M}$.
- Mean translational KE per molecule = $(3/2)kT$, so T measures average KE.
- $PV = NkT = nRT$; $R = N_A k = 8.314 \text{ J mol}^{-1} \text{ K}^{-1}$; $k = 1.38 \times 10^{-23} \text{ J K}^{-1}$.
- Boyle: $PV = \text{const}$; Charles: $V \propto T$; Gay-Lussac: $P \propto T$; Avogadro: equal volumes, equal molecules; Dalton: $P = \sum P_i$; Graham: $\text{rate} \propto 1/\sqrt{M}$.

REVISION · Formulae and questions hours

Revision section

A. Formula sheet (all units)

Topic	Formula
Equivalent focal length of two thin lenses	$1/F = 1/f_1 + 1/f_2 - d/(f_1 f_2)$
Huygens and Ramsden eyepieces	Huygens: $f_1 = 3f, f_2 = f, d = 2f, F = 3f/2$. Ramsden: $f_1 = f_2 = f, d = 2f/3, F = 3f/4$
Compound microscope	$M = (v_o/u_o)(1 + D/f_e)$; normal adjustment $M \approx (L/f_o)(D/f_e)$
Resolving power of microscope	$d_{\min} = 0.61\lambda/(n \sin \theta)$
Astronomical telescope	$M = f_o/f_e$ (normal adjustment); $L = f_o + f_e$; $M = (f_o/f_e)(1 + f_e/D)$ for image at D
Faraday's law	$\epsilon = -N d\Phi/dt$; motional emf $\epsilon = Blv$
Self-inductance	$\epsilon = -L di/dt$; $U = \frac{1}{2}LI^2$; solenoid $L = \mu_0 N^2 A/l$
Mutual inductance	$\epsilon_2 = -M di_1/dt$; coaxial solenoids $M = \mu_0 N_1 N_2 A/l$; $M = k\sqrt{L_1 L_2}$
Series and parallel resistance	$R_s = \Sigma R$; $1/R_p = \Sigma 1/R$
Thevenin, Norton	$I_N = V_{Th}/R_{Th}$; $R_N = R_{Th}$; $I_L = V_{Th}/(R_{Th} + R_L)$
Maximum power transfer	$R_L = R_{Th}$; $P_{\max} = V_{Th}^2/(4R_{Th})$; efficiency 50 %
Kinetic theory	$P = (1/3)\rho\langle c^2 \rangle$; $c_{\text{rms}} = \sqrt{3RT/M}$; KE per molecule = $(3/2)kT$; $PV = nRT$

B. Short-answer questions (2–3 marks each)

1. Name two defects of a single lens that are reduced in a compound eyepiece.
2. Why is a Ramsden eyepiece preferred to a Huygens eyepiece in measuring instruments?
3. Why is the separation of the lenses of a Ramsden eyepiece made $2f/3$ and not f ?
4. Write the expression for magnifying power of an astronomical telescope in normal adjustment.
5. State two advantages of a reflecting telescope over a refracting telescope.
6. What is the principle of a constant deviation spectrometer?
7. State Faraday's laws of electromagnetic induction.
8. State Lenz's law and show that it follows from conservation of energy.
9. Define the henry. On what factors does the self-inductance of a solenoid depend?
10. What is the coupling coefficient between two coils? What is its range?
11. State Kirchhoff's current and voltage laws.
12. State the superposition theorem. For which type of circuits is it valid?
13. What is meant by the Thevenin resistance? How is it determined?

14. Why is the efficiency 50 % at maximum power transfer?
15. List four postulates of the kinetic theory of gases.
16. Define the root mean square speed of gas molecules.
17. State Dalton's law of partial pressures and Graham's law of diffusion.

C. Long-answer questions (5–8 marks each)

1. With a neat diagram describe the construction and working of a Huygens eyepiece. Show that it satisfies the condition for achromatism.
2. Describe the Ramsden eyepiece. Compare it with the Huygens eyepiece.
3. Explain the working of a compound microscope with a ray diagram. Derive an expression for its magnifying power.
4. Describe an astronomical telescope. Derive its magnifying power in normal adjustment and obtain the length of the tube.
5. Describe the construction of a reflecting telescope (Newtonian or Cassegrain) and discuss its advantages and disadvantages.
6. Explain the constant deviation spectrometer. What is the Pellin–Broca prism, and what are its advantages?
7. State Faraday's laws and Lenz's law. Derive the expression for the motional emf in a conductor moving in a magnetic field.
8. Define self-inductance. Derive an expression for the self-inductance of a long solenoid and obtain the energy stored in an inductor.
9. Define mutual inductance. Derive the expression for the mutual inductance of (a) two co-axial solenoids and (b) two co-axial circular coils.
10. State and explain Kirchhoff's laws. Use them to find the currents in a network of two cells and three resistors.
11. State and prove the maximum power transfer theorem.
12. State Thevenin's and Norton's theorems with one numerical example, and show how the two equivalents are related.
13. Derive an expression for the pressure exerted by an ideal gas on the basis of the kinetic theory.
14. Deduce Boyle's law and the gas equation $PV = nRT$ from the kinetic theory of gases.
15. Derive Charles's law, Avogadro's law and Graham's law of diffusion from the kinetic equation of a gas.

D. Multiple-choice questions

1. In a Huygens eyepiece the ratio of the focal lengths of field lens to eye lens is (a) 1:1 (b) 2:1 (c) 3:1 (d) 1:3.
2. The separation between the lenses of a Ramsden eyepiece is (a) f (b) $2f$ (c) $2f/3$ (d) $f/2$.
3. A telescope has objective of focal length 80 cm and eyepiece of focal length 4 cm. In normal adjustment its magnifying power is (a) 20 (b) 76 (c) 84 (d) 320.
4. The collimator and telescope of a constant deviation spectrometer are fixed at (a) 45° (b) 60° (c) 90° (d) 180° .
5. The unit of magnetic flux is (a) tesla (b) weber (c) henry (d) gauss.

6. The negative sign in Faraday's law is a consequence of (a) Ohm's law (b) Lenz's law (c) Ampère's law (d) Coulomb's law.
7. The self-inductance of a solenoid is proportional to (a) N (b) N^2 (c) $1/N$ (d) N^3 .
8. If the coupling between two coils is perfect, then (a) $M = 0$ (b) $M = L_1 + L_2$ (c) $M = \sqrt{L_1 L_2}$ (d) $M = L_1 L_2$.
9. Kirchhoff's voltage law is based on conservation of (a) charge (b) energy (c) momentum (d) mass.
10. Three resistors $6\ \Omega$ each are connected in parallel. The equivalent resistance is (a) $18\ \Omega$ (b) $6\ \Omega$ (c) $3\ \Omega$ (d) $2\ \Omega$.
11. The Norton current of a source is (a) open-circuit current (b) short-circuit current (c) load current (d) zero.
12. Maximum power is transferred to a load when R_L is (a) 0 (b) infinite (c) R_{Th} (d) $2R_{Th}$.
13. The r.m.s. speed of gas molecules is proportional to (a) T (b) $\sqrt{T}$ (c) $1/T$ (d) T^2 .
14. The mean translational kinetic energy per molecule of an ideal gas is (a) kT (b) $(3/2)kT$ (c) $(1/2)kT$ (d) $3kT$.
15. At constant temperature, the product PV of a gas is (a) increasing (b) decreasing (c) constant (d) zero.
16. In the same conditions, the ratio of r.m.s. speeds of H_2 to O_2 molecules is (a) 1 (b) 4 (c) 16 (d) $1/4$.

Answers: 1-c, 2-c, 3-a, 4-c, 5-b, 6-b, 7-b, 8-c, 9-b, 10-d, 11-b, 12-c, 13-b, 14-b, 15-c, 16-b.

E. Numerical practice problems

1. The focal lengths of the objective and eyepiece of a microscope are 1 cm and 5 cm; the tube length is 15 cm. Find the magnification for the final image at infinity ($D = 25$ cm).
2. A telescope with objective of focal length 120 cm gives a magnification of 30 in normal adjustment. Find the focal length of the eyepiece and the length of the tube.
3. The magnetic flux through a 100-turn coil changes from 4×10^{-3} Wb to zero in 0.05 s. Find the induced emf.
4. Find the self-inductance of a solenoid of 1000 turns, length 0.5 m and radius 2 cm ($\mu_0 = 4\pi \times 10^{-7}$ H m $^{-1}$). Also find the energy stored when it carries 2 A.
5. Two coils have self-inductances 4 mH and 9 mH, and the coupling coefficient is 0.5. Find the mutual inductance.
6. In the network of Figure 3.1 change E_1 to 18 V and keep all else the same; find the current in R_3 using superposition.
7. A battery of emf 10 V and internal resistance $2\ \Omega$ is connected to a load. Find the load resistance for maximum power and the maximum power.
8. Find the r.m.s. speed of nitrogen molecules ($M = 28$ g mol $^{-1}$) at 27°C .
9. At what temperature will the r.m.s. speed of hydrogen molecules be equal to that of oxygen molecules at 300 K?

Answers and hints

1. $m_o = L/f_o = 15$; $m_e = D/f_e = 5$; $M = 15 \times 5 = 75$.
2. $f_e = 120/30 = 4$ cm; $L = 124$ cm.
3. $\epsilon = 100 \times 4 \times 10^{-3}/0.05 = 8$ V.

4. $A = \pi(0.02)^2 = 1.257 \times 10^{-3} \text{ m}^2$; $L = \mu_0 N^2 A / l = 4\pi \times 10^{-7} \times 10^6 \times 1.257 \times 10^{-3} / 0.5 \approx 3.16 \times 10^{-3} \text{ H}$;
 $U = \frac{1}{2} LI^2 = 6.3 \times 10^{-3} \text{ J}$.
5. $M = k\sqrt{L_1 L_2} = 0.5 \times \sqrt{4 \times 9} \text{ mH} = 3 \text{ mH}$.
6. E_1 alone gives $(18/4) \times 3/9 = 1.5 \text{ A}$; E_2 alone gives 0.333 A ; $I_3 = 1.83 \text{ A}$.
7. $R_L = 2 \Omega$; $P_{\max} = 10^2 / (4 \times 2) = 12.5 \text{ W}$.
8. $c = \sqrt{3 \times 8.314 \times 300 / 0.028} \approx 517 \text{ m s}^{-1}$.
9. $c \propto \sqrt{T/M}$: $T_H/2 = 300/32$, so $T = 300 \times 2/32 \approx 18.8 \text{ K}$.